Application of Normal, Gamma and Extreme Value Families of Probability Distributions in Frequency Analysis of Rainfall and Stream Flow Data

Technical Session

Parallel Session – PW5

Session Theme

Extreme Events and their Management

(Key Note Lecture)

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8th International R&D Conference on Global Trends in Water Resources, Power & RE Sectors during 8-9 February 2024 at IIT Roorkee

Effect of Data Length on Estimation of Rainfall using Six Probability Distributions

Outline of the Presentation

- Introduction
- Methodology

  Extreme Value Analysis (EVA) of Rainfall)

  - Normal family : Log Normal (LN2)
  - Gamma family: Log Pearson Type-3 (LP3)
  - Extreme value family : Extreme Value Type-1 (EV1), Extreme Value Type-2 (EV2), Generalized Extreme Value (GEV), Generalized Pareto (PR3)
  - Goodness-of-Fit (GoF) tests viz., Chi-Square (χ^2) & Kolmogorov-Smirnov (KS) and diagnostic test viz., Correlation Coefficient (CC) and D-index.
- Study area and data used
- Results and Discussion
- Recommendation

Introduction

- Design rainfall is one of the hydrology quantities that can be used for computing the design flood.
- Design rainfall depth can also be applied to design and manage civil engineering structures viz., water use and flood control structures, and drainage channels in urban areas.
- Frequency analysis of rainfall is considered as one of the important tools to estimate the design rainfall.
- Dimensions and capacity of the buildings, in addition to being determined by the service volume, also depend on the design flood that corresponds to the selected return period.
- 2-parameter Log Normal (LN2), Log Pearson Type-3 (LP3), Extreme Value Type-1 (EV1), Extreme Value Type-2 (EV2), Generalized Extreme Value (GEV) and 3-parameters Pareto (PR3) distributions are applied in frequency analysis of rainfall data.

Methodology

Probability Distributions used in Rainfall Frequency Analysis

Probability distribution	Cumulative Distribution Function (F(x) or F(y))	Quantile estimator (x(T))
LN2 ($\mu(y)$, $\sigma(y)$)	$F(y) = \varphi\left(\frac{y-\mu(y)}{\sigma(y)}\right)$,y>0 with y=ln(x)	$x(T) = \exp\left(\mu(y) + \sigma(y)K(T)\right)$; φ^{-1} : Inverse of standard normal distribution
LP3 (ξ, α, k)	$F(x) = \begin{cases} G\left(k, \frac{\ln(x)-\xi}{\alpha}\right) & ,\alpha > 0 \\ 1 - G\left(k, \frac{\ln(x)-\xi}{\alpha}\right) & ,\alpha < 0 \end{cases}$	No explicit expression of the quantile function is available
GEV (ξ, α, k)	$F(x) = \exp\left(-\left[1 - \frac{k(x-\xi)}{\alpha}\right]^{1/k}\right)$ x>0, α>0	$x(T) = \xi + \frac{\alpha}{k}\left(1 - \left[-\ln(1-(1/T))\right]^{k}\right)$
EV1 (ξ, α)	$F(x) = \exp\left(-\exp-\left(\frac{x-\xi}{\alpha}\right)\right)$ x>0, α>0	$x(T) = \xi - \alpha \ln\left[-\ln(1-(1/T))\right]$
EV2 (α, k)	$F(x) = \exp\left(-\left(\frac{x}{\alpha}\right)^{-k}\right)$ x>0, α>0, k>0	$x(T) = \alpha\left[-\ln\left(1-(1/T)\right)\right]^{-(1/k)}$
PR3 (ξ, α, k)	$F(x) = 1 - \left[1 - \frac{k(x-\xi)}{\alpha}\right]^{\frac{1}{k}}$ x>0, α>0, k>0	$x(T) = \xi + \frac{\alpha\left[1-(1/T)^{k}\right]}{k}$

ξ : Location parameter	α : Scale parameter	k : Scale parameter
G(…):Incomplete gamma integral	$\mu(y)$: Average of log-transformed series of observed data	T: Return period
F(x) : Cumulative distribution function of x	$\sigma(y)$: Standard deviation of log-transformed series of observed data	K(T): Frequency factor for a given return period

Methodology

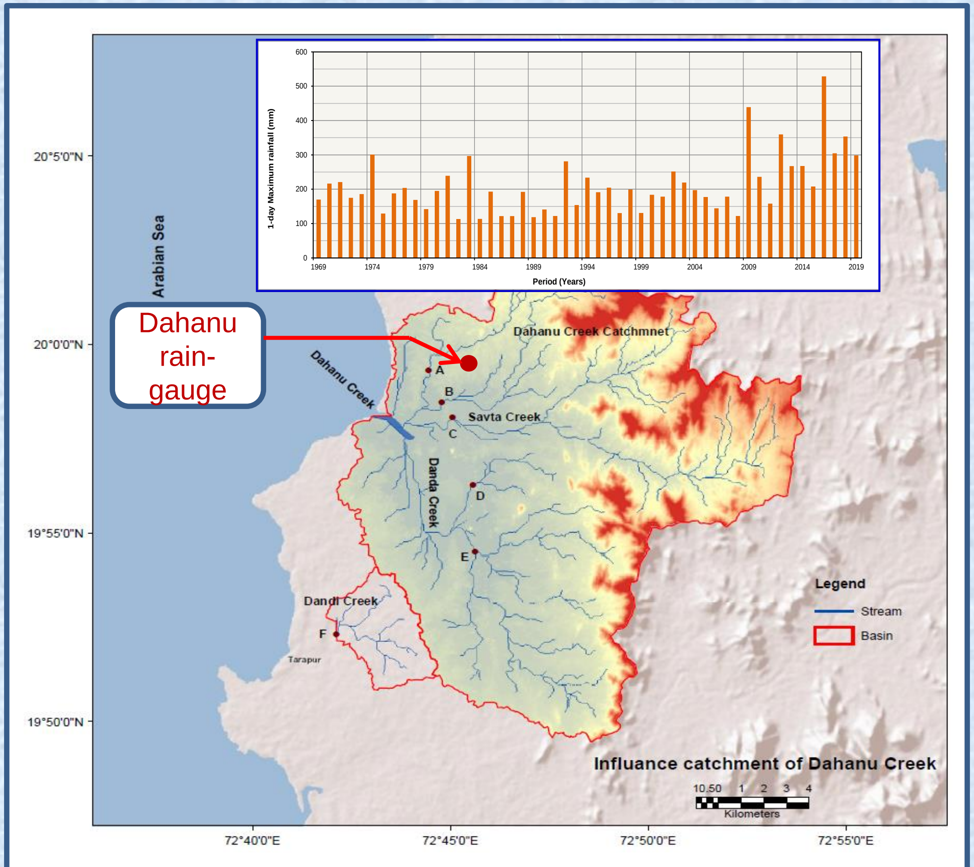
Goodness-of-Fit and Diagnostic Tests

Goodness-of-Fit (GoF) Tests		Diagnostic Test	
$\chi^2 = \sum_{j=1}^{NC} \frac{\left(O_j(q) - E_j(q)\right)^2}{E_j(q)}$	$KS = \underset{i=1}{\overset{N}{Max}}\left(F_e(x(i)) - F_D(x(i))\right)$	$CC = \frac{\sum_{i=1}^{N}\left(x(i)-\mu(x)\right)\left(x^*(i)-\mu(x^*)\right)}{\sqrt{\left(\sum_{i=1}^{N}\left(x(i)-\mu(x)\right)^2\right)\left(\sum_{i=1}^{N}\left(x^*(i)-\mu(x^*)\right)^2\right)}}$	$D-index = \left(1/\mu(x)\right)\sum_{i=1}^{6}\left\lvert x(i) - x^*(i)\right\rvert$

Goodness-of-Fit Tests: χ^2 : Chi-Square KS : Kolmogorov-Smirnov		
$O_j(q)$	Observed frequency value of q for j^{th} class	
$E_j(q)$	Expected frequency value of q for j^{th} class	
NC	Number of frequency classes	
CDF	Cumulative Distribution Function	
$F_e(x(i))$	Empirical CDF of x(i) of i^{th} sample [$F_e(x(i))=r/(N+1)$]	
$F_D(x(i))$	Derived CDF of x(i) of i^{th} sample by probability distribution function	
x(i)	Observed value of x of i^{th} sample	
Diagnostic test		
x(i)	Observed value of x of i^{th} sample (first six highest values in the observed data series)	
$x^*(i)$	Estimated value of i^{th} sample (predicted values by probability distribution function)	
$\mu(x)$	Average of observed values	$\mu(x^*)$: Average of predicted values
N	Total number of samples	

- If the computed values of GoF tests statistic by the distribution are not greater than its theoretical values at either 5% or 1% level of significance then the distribution is considered to be acceptable for extreme value analysis at the level.
- Probability distribution with least D-index is found to be best for rainfall estimation.

Study Area and Data Used



- Dahanu site is located in North Konkan region between the latitude 19.98 °N and longitude 72.74 °E and at a distance of 130 km from Mumbai.
- Annual 1-day Maximum Rainfall (AMR) series derived from the daily rainfall data observed at Dahanu for the period 1970 to 2019 is used to create three rainfall series with different data length viz.,
  - D30 with 30 years data (1970 to 1999),
  - D40 with 40 years data (1970 to 2009),
  - D50 with 50 years data (1970 to 2019),

  which are applied in estimating the extreme (i.e., 1-day maximum) rainfall using six probability distributions (viz., LN2, LP3, EV1, EV2, GEV and PR3).

Data series	Original series				Log-transformed series			
	Ave. (mm)	SD (mm)	C_s	C_k	Ave. (mm)	SD (mm)	C_s	C_k
D30	180.8	53.9	0.662	-0.151	5.156	0.292	0.158	-0.935
D40	187.9	64.1	1.715	4.882	5.187	0.307	0.536	0.468
D50	209.9	84.7	1.618	3.543	5.280	0.360	0.519	0.084

Ave.: Average; SD: Standard Deviation; C_s: Coefficient of Skewness; C_k: Coefficient of Kurtosis.

Estimated 1-day maximum rainfall given by D30 series using six distributions

Probability Distribution	Method	1-day maximum rainfall (mm) for different return periods (year)							
		2	**5**	**10**	**20**	**25**	**50**	**75**	**100**
LN2	MoM	173.3	221.5	251.8	279.9	288.7	315.4	330.7	341.5
	MLM	173.4	220.9	250.7	278.2	286.8	313.0	327.9	338.5
	LMO	173.4	223.1	254.4	283.7	289.5	320.6	333.9	347.8
LP3	MoM	173.4	221.8	252.2	280.5	289.3	316.1	331.5	342.3
	MLM	171.6	220.0	252.0	282.8	292.7	322.9	340.8	353.5
	LMO	NE	NE	NE	NE	NE	NE	NE	NE
EV1	MoM	171.9	219.6	251.1	281.4	290.9	320.5	337.7	349.8
	MLM	171.4	219.1	250.6	280.9	290.5	320.0	337.2	349.4
	LMO	171.5	221.3	254.3	285.9	295.9	326.8	344.7	357.5
EV2	MoM	167.9	208.7	241.0	276.7	289.1	330.9	357.9	378.4
	MLM	165.1	219.7	265.4	318.1	336.9	402.2	445.9	479.6
	LMO	164.6	212.2	251.2	295.2	310.7	363.9	398.9	425.7
GEV	MoM	174.7	223.3	252.7	279.0	287.0	310.6	323.5	332.4
	MLM	172.8	219.6	249.2	276.6	285.1	310.8	325.3	335.4
	LMO	172.6	222.3	254.2	284.0	293.3	321.5	337.6	348.9
PR3	MoM	170.5	229.6	260.9	284.0	290.0	305.4	312.5	316.8
	MLM	134.5	152.7	159.4	163.0	163.8	165.4	166.0	166.3
	LMO	171.3	230.6	260.9	282.6	288.2	302.1	308.3	312.0

Estimated 1-day maximum rainfall given by D40 series using six distributions

Probability Distribution	Method	1-day maximum rainfall (mm) for different return periods (year)							
		2	5	10	20	25	50	75	100
LN2	MoM	177.8	235.1	272.1	307.0	318.0	351.7	371.2	385.0
	MLM	179.0	231.0	264.0	294.7	304.3	333.6	350.5	362.3
	LMO	179.0	231.5	264.8	295.9	302.2	335.3	349.6	364.5
LP3	MoM	174.2	229.2	268.9	309.4	322.8	365.9	392.2	411.4
	MLM	172.3	227.6	269.5	313.8	328.7	377.5	408.0	430.6
	LMO	NE	NE	NE	NE	NE	NE	NE	NE
EV1	MoM	177.3	234.1	271.6	307.6	319.0	354.2	374.7	389.2
	MLM	177.2	227.7	261.2	293.3	303.5	334.9	353.1	366.0
	LMO	177.8	232.3	268.4	303.0	314.0	347.8	367.4	381.4
EV2	MoM	172.3	219.4	257.4	300.1	315.1	366.1	399.5	424.9
	MLM	170.2	228.2	277.0	333.6	353.9	424.4	471.7	508.3
	LMO	169.7	221.7	264.6	313.5	330.8	390.5	429.9	460.3
GEV	MoM	175.0	229.3	268.1	307.5	320.5	362.0	387.3	405.6
	MLM	173.0	227.1	268.2	311.9	326.8	375.8	406.7	429.7
	LMO	174.9	229.2	268.0	307.4	320.4	362.1	387.4	405.8
PR3	MoM	171.8	230.1	274.5	317.2	330.6	371.4	394.5	410.6
	MLM	213.3	259.8	294.3	328.1	338.8	371.8	390.9	404.2
	LMO	173.0	237.7	277.0	309.6	318.9	344.4	357.2	365.5

Estimated 1-day maximum rainfall given by D50 series using six distributions

Probability Distribution	Method	1-day maximum rainfall (mm) for different return periods (year)							
		2	5	10	20	25	50	75	100
LN2	MoM	194.7	269.9	320.2	368.7	384.2	432.2	460.3	480.4
	MLM	196.3	265.0	310.0	352.8	366.4	408.2	432.5	449.8
	LMO	196.3	266.0	311.8	355.6	363.2	412.2	435.0	454.8
LP3	MoM	190.3	262.4	316.2	372.4	391.3	452.6	490.7	518.8
	MLM	187.9	260.3	316.8	377.8	398.7	468.0	512.0	544.9
	LMO	NE	NE	NE	NE	NE	NE	NE	NE
EV1	MoM	196.0	270.9	320.4	368.0	383.1	429.5	456.5	475.6
	MLM	195.4	260.3	303.3	344.6	357.7	398.0	421.4	438.0
	LMO	196.5	268.8	316.7	362.6	377.2	422.1	448.2	466.6
EV2	MoM	189.0	248.3	297.6	354.0	374.0	443.1	489.0	524.4
	MLM	185.0	260.6	327.0	406.5	435.6	538.8	609.6	665.4
	LMO	184.2	251.7	309.6	377.6	402.1	488.2	546.5	591.9
GEV	MoM	193.3	265.6	316.6	368.1	385.0	438.6	470.9	494.4
	MLM	189.9	258.0	309.2	363.5	381.8	442.1	479.9	508.0
	LMO	189.9	261.1	315.0	372.4	391.9	456.2	496.7	526.8
PR3	MoM	185.1	267.2	325.6	381.0	398.3	450.1	479.1	499.2
	MLM	228.2	293.5	341.4	388.1	402.9	448.0	473.8	491.9
	LMO	187.1	271.7	327.8	378.1	393.1	436.5	459.8	475.4

GoF and Diagnostic tests statistic values computed by six distributions

PDF	Method	Computed values of GoF and diagnostic tests statistic for											
		D30				D40				D50			
		χ^2	KS	CC	D-index	χ^2	KS	CC	D-index	χ^2	KS	CC	D-Index
LN2	MoM	5.200	0.121	0.926	0.363	10.40	0.089	0.946	0.767	9.200	0.062	0.989	0.913
	MLM	5.200	0.121	0.927	0.384	10.40	0.089	0.945	0.874	9.200	0.062	0.989	1.162
	LMO	5.210	0.125	0.927	0.377	10.42	0.092	0.844	0.997	9.215	0.067	0.983	1.240
LP3	MoM	5.200	0.122	0.926	0.356	9.600	0.114	0.952	0.693	10.80	0.074	0.991	0.736
	MLM	5.200	0.122	0.922	0.332	9.600	0.114	0.955	0.645	10.80	0.074	0.991	0.731
	LMO	NE	NE	NE	NE	NE	NE	NE	NE	NE	NE	NE	NE
EV1	MoM	7.200	0.131	0.923	0.338	8.400	0.092	0.947	0.753	8.800	0.068	0.989	0.937
	MLM	5.760	0.127	0.923	0.345	7.920	0.090	0.947	0.872	7.700	0.068	0.989	1.322
	LMO	3.600	0.122	0.923	0.330	7.200	0.088	0.947	0.771	6.000	0.068	0.989	1.011
EV2	MoM	12.40	0.210	0.905	0.601	15.60	0.162	0.959	0.748	15.20	0.154	0.991	1.069
	MLM	8.880	0.200	0.899	0.442	15.44	0.158	0.962	0.527	14.70	0.134	0.991	0.360
	LMO	10.10	0.184	0.901	0.504	15.20	0.152	0.960	0.578	14.09	0.104	0.991	0.511
GEV	MoM	13.12	0.116	0.931	0.381	8.000	0.108	0.952	0.708	8.800	0.059	0.990	0.862
	MLM	9.040	0.115	0.927	0.408	7.680	0.109	0.956	0.650	9.600	0.064	0.991	0.883
	LMO	17.20	0.114	0.926	0.322	7.200	0.109	0.952	0.707	10.80	0.072	0.991	0.709
PR3	MoM	4.000	0.108	0.950	0.408	11.60	0.152	0.945	0.673	11.80	0.108	0.988	0.796
	MLM	4.320	0.106	0.964	0.434	12.56	0.140	0.947	1.072	11.80	0.105	0.989	0.970
	LMO	4.160	0.131	0.923	0.338	14.00	0.121	0.930	0.801	12.40	0.101	0.986	0.902

Theoretical values of χ^2 statistic at 5% level of significance with degrees of freedom (NC-m-1) viz., 2, 3, 4, 5, 6, 7 are 5.991, 7.815, 9.488, 11.070, 12.592 and 14.067 respectively; Theoretical values of KS statistic at 5% level of significance with sample values viz., 30, 40 and 50 are 0.242, 0.210 and 0.188 respectively.

Evaluation of Results Using GoF Tests

i) χ^2 test results confirmed the applicability of LN2, LP3, EV1 and PR3 distributions for rainfall estimation using D30 series.

ii) χ^2 test results didn’t confirm the applicability of LP3, EV2 and PR3 distributions for estimating the rainfall using D40 series.

iii) χ^2 test results showed that the LN2, LP3, EV1, GEV and PR3 distributions are adequate in rainfall estimation using D50 series.

iv) KS tests results supported the use of all six probability distributions adopted in estimating the rainfall using D30, D40 and D50 series.

Evaluation of Results Using GoF Tests

i) For D30, D40 and D50 series, it is noted that the CC values computed by six probability distributions vary from 0.899 to 0.964 for D30 series while 0.844 to 0.962 for D40 series and 0.983 to 0.991 for D50 series.

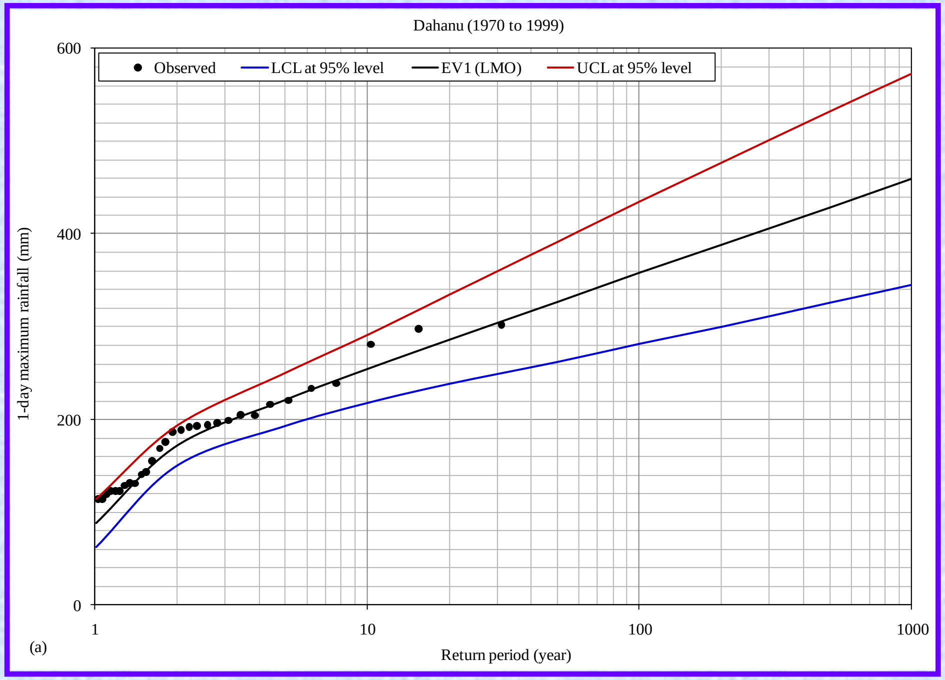
ii) D-index values of GEV (LMO) for D30, EV2 (MLM) for D40 and EV2 (MLM) for D50 are noted to be minimum than those values computed by other distributions adopted in rainfall estimation.

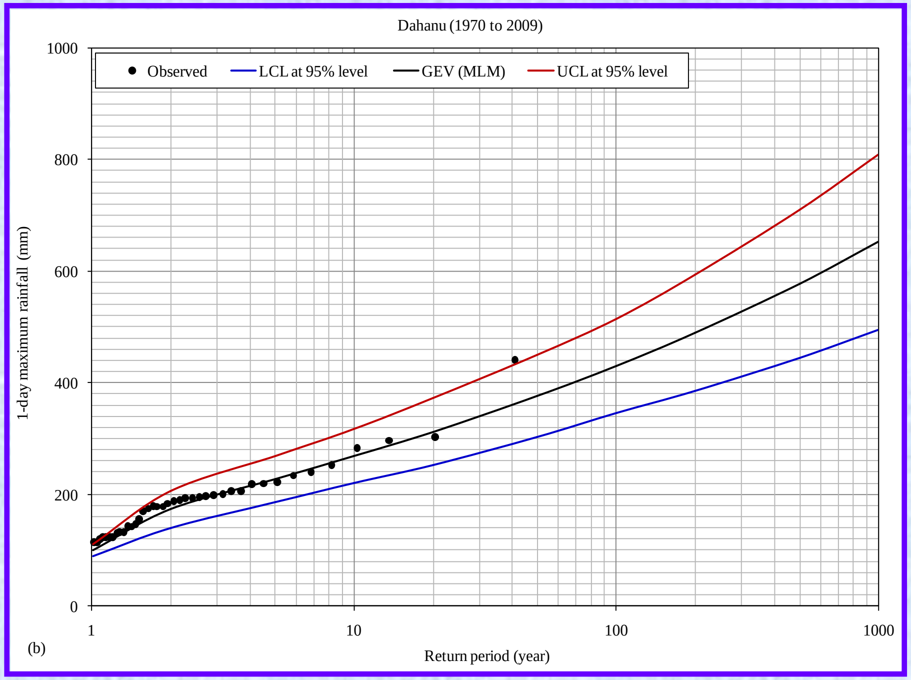
Selection of Probability Distribution

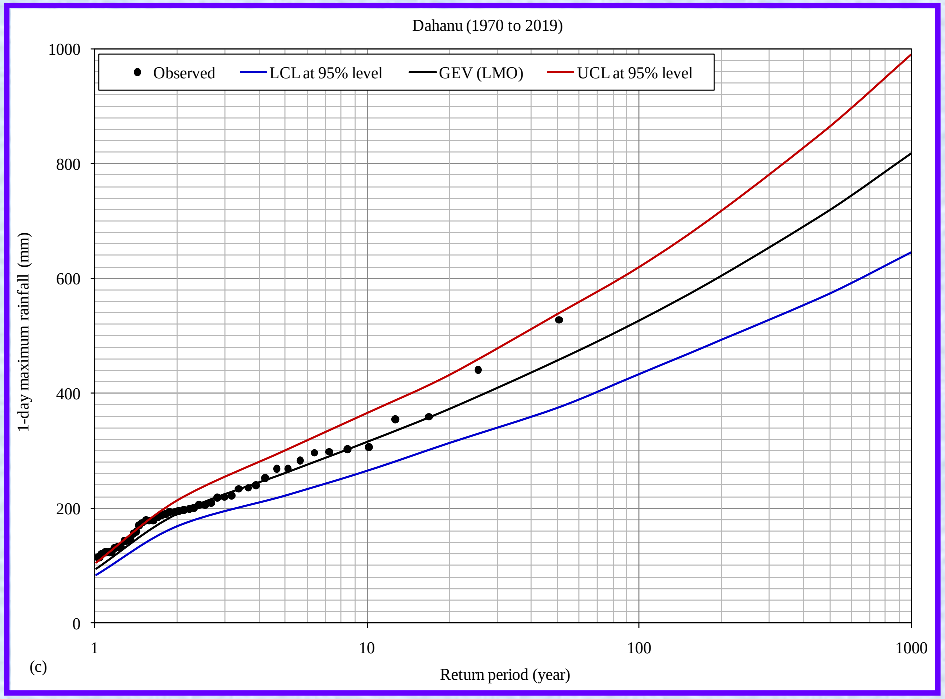
i) D-index given by EV1 (LMO) is the second minimum next to GEV (LMO) for D30 while the D-index of EV2 (LMO), LP3 (MLM) and GEV (MLM) are the second, third and fourth minimum next to EV2 (MLM) for D40. Likewise, the D-index of EV2 (LMO) and GEV (LMO) are the second and third minimum next to EV2 (MLM).

i) However, χ^2 test didn't confirm the adequacy of fitting GEV (LMO) distribution for D30 series while EV2 (using MLM and LMO) and LP3 (MLM) for D40, and EV2 (using MLM and LMO) for D50.

i) In light of the above, after eliminating the D-index values given by GEV (LMO) for D30, EV2 (using MLM and LMO) and LP3 (MLM) for D40, and EV2 (using MLM and LMO) for D50, it is identified that the EV1 (LMO) is better suited distribution for rainfall estimation while using the D30 series whereas GEV (MLM) for D40 series and GEV (LMO) for D50 series.







- Estimated rainfall for different return periods using D30, D40 and D50 series are in increasing order when data length increases.

- The percentages of observed AMR are lying within the confidence limits of the estimated 1-day maximum rainfall using D30, D40 and D50 series are about 98%, 95% and 85% respectively.

Conclusions

- Study on effect of data length on estimation of rainfall using D30, D40 and D50 series of Dahanu by adopting six probability distributions (viz., LN2, LP3, EV1, EV2, GEV and PR3) was carried out.
- Adequacy of fitting distributions to the rainfall series was evaluated by quantitative assessment using GoF (viz., χ^2 and KS) and diagnostic (viz., CC and D-index) tests.
- χ^2 test results confirmed the applicability of LN2, LP3, EV1 and PR3 distributions for rainfall estimation using D30 and D50 series.
- χ^2 test results supported the use of LN2, EV1 and GEV distributions for estimating the rainfall using D40 series.
- KS tests results supported the use of all six probability distributions adopted in estimating the rainfall using D30, D40 and D50 series.
- Quantitative assessment of the outcomes was weighed with GoF tests and D-index values and accordingly EV1 (LMO) is identified as better suited distribution for rainfall estimation while using D30 series whereas GEV (MLM) for D40 series and GEV (LMO) for D50 series.

Intercomparison of Peak Flood Discharge Estimates using Normal, Gamma and Extreme Value Families of Probability Distributions

Outline of the Presentation

- Introduction
- Methodology

  Extreme Value Analysis (EVA) of Rainfall)

  - ❖Normal family : Normal (N2), Log Normal (LN2)
  - ❖Gamma family: Gamma (GAM), Exponential (EXP)
    Log Pearson Type-3 (LP3)
  - ❖Extreme value family : Extreme Value Type-1 (EV1), Extreme Value Type-2 (EV2), Generalized Extreme Value (GEV)
  - ❖Goodness-of-Fit (GoF) tests viz., Chi-Square (χ^2) & Kolmogorov-Smirnov (KS) and diagnostic test (D-index).
- Study area and data used
- Results and Discussion
- Recommendation

Introduction

- Floods are natural phenomena that cause disasters like destruction of infrastructure, damages in environmental and agricultural lands, mortality and economic losses.
- Flooding is the most common hazard among the environmental hazards that impose damage in commercial buildings, roads, bridges, water supply systems and sewage disposal, agricultural lands, etc.
- There is a need to estimate the flood magnitude for a given return period at a particular site. This can be achieved by conducting flood frequency analysis using stream flow data.
- These flood estimates have been used for various purposes including planning and design of hydraulic structures such as barrages, canals, bridges, dams, embankments, reservoirs and spillways, insurance studies, planning of flood management and rescue operations.

Methodology

Probability Distributions used in Flood Frequency Analysis

Family	PDF	CDF [F(x) of F(y)]	Quantile estimator (x(T))
Normal	NOR (μ(x), σ(x))	$F(x) = \varphi\left(\frac{x-\mu(x)}{\sigma(x)}\right), 0<x<\infty, \sigma(x)>0$	$x(T) = \mu(x) + \sigma(x)\,\varphi^{-1}(1/T)$ — φ^{-1}: Inverse of standard normal distribution
	LN2 (μ(y), σ(y))	$F(y) = \varphi\left(\frac{y-\mu(y)}{\sigma(y)}\right), 0<y<\infty, \sigma(y)>0$	$x(T) = \exp\,(\mu(y) + \sigma(y)\,\varphi^{-1}(1/T))$ — with y=ln(x)
Gamma	GAM (α, k)	$F(x) = \begin{cases} G\left(k, \frac{x}{\alpha}\right) & ,\alpha>0 \\ 1-G\left(k, \frac{x}{\alpha}\right) & ,\alpha<0 \end{cases}$	No explicit expression of quantile function is available.
	EXP (ξ, α)	$F(q) = 1-\exp\left[-\frac{(q-\xi)}{\alpha}\right]$	$x(T) = \xi - \alpha \ln(1/T)$
	LP3 (ξ, α, k)	$F(x) = \begin{cases} G\left(k, \frac{\ln(x)-\xi}{\alpha}\right) & ,\alpha>0 \\ 1-G\left(k, \frac{\ln(x)-\xi}{\alpha}\right) & ,\alpha<0 \end{cases}$	No explicit expression of quantile function is available.
Extreme Value	EV1 (ξ, α)	$F(x) = \exp\left(-\exp-\left(\frac{x-\xi}{\alpha}\right)\right), \alpha>0, -\infty<x<\infty$	$x(T) = \xi - \alpha \ln[-\ln(1-(1/T))]$
	EV2 (α, k)	$F(x) = \exp\left(-\left(\frac{x}{\alpha}\right)^{-k}\right), \alpha>0, k>0, 0<x<\infty$	$x(T) = \alpha\,[-\ln(1-(1/T))]^{-(1/k)}$ — x(T): Estimated peak flood discharge by probability distribution
	GEV (ξ, α, k)	$F(x) = \exp\left(-\left[1-\frac{k(x-\xi)}{\alpha}\right]^{1/k}\right), -\infty \le x \le \xi+(\alpha/k) \text{ for } k>0,\ \alpha>0$	$x(T) = \xi + \frac{\alpha}{k}\left(1-[-\ln(1-(1/T))]^{k}\right)$
ξ : Location parameter		α : Scale parameter	k : Scale parameter
G(…):Incomplete gamma integral		μ(x): Average of observed data (x)	μ(y): Average of log-transformed series of observed data
F(x) : Cumulative distribution function of x		σ(x): Standard deviation of observed data (x)	σ(y): Standard deviation of log-transformed series of observed data

K(T): Frequency factor for a return period (T) corresponding to probability of exceedance (P) and Coefficient of Skewness (C_s) (i.e., C_s=0.0 for LN2 and C_s of the observed data for PR3)

Methodology

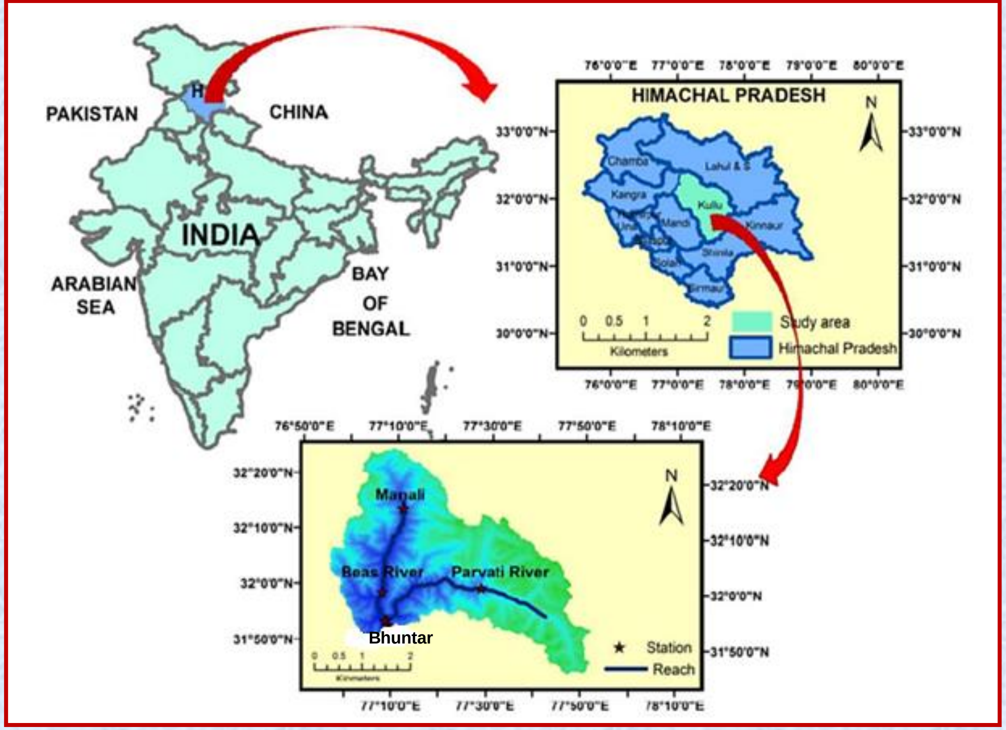
Goodness-of-Fit and Diagnostic Tests

Goodness-of-Fit (GoF) Tests		Diagnostic Test
$\chi^2 = \sum_{j=1}^{NC} \frac{(O_j(q) - E_j(q))^2}{E_j(q)}$	$KS = \underset{i=1}{\overset{N}{Max}}(F_e(x(i)) - F_D(x(i)))$	$D-index = (1/\mu(x)) \sum_{i=1}^{6} \lvert x(i) - x^*(i) \rvert$

Goodness-of-Fit Tests: χ^2 : Chi-Square **KS** : Kolmogorov-Smirnov	
$O_j(q)$	Observed frequency value of q for j^{th} class
$E_j(q)$	Expected frequency value of q for j^{th} class
NC	Number of frequency classes
CDF	Cumulative Distribution Function
$F_e(x(i))$	Empirical CDF of x(i) of i^{th} sample [$F_e(x(i))=r/(N+1)$]
$F_D(x(i))$	Derived CDF of x(i) of i^{th} sample by probability distribution function
x(i)	Observed value of x of i^{th} sample
Diagnostic test	
x(i)	Observed value of x of i^{th} sample (first six highest values in the observed data series)
$x^*(i)$	Estimated value of i^{th} sample (predicted values by probability distribution function)
$\mu(x)$	Average of observed values
N	Total number of samples

- If the computed values of GoF tests statistic by the distribution are not greater than its theoretical values at either 5% or 1% level of significance then the distribution is considered to be acceptable for flood frequency analysis at the level.
- Probability distribution with least D-index is considered as best for estimation of peak flood discharge.

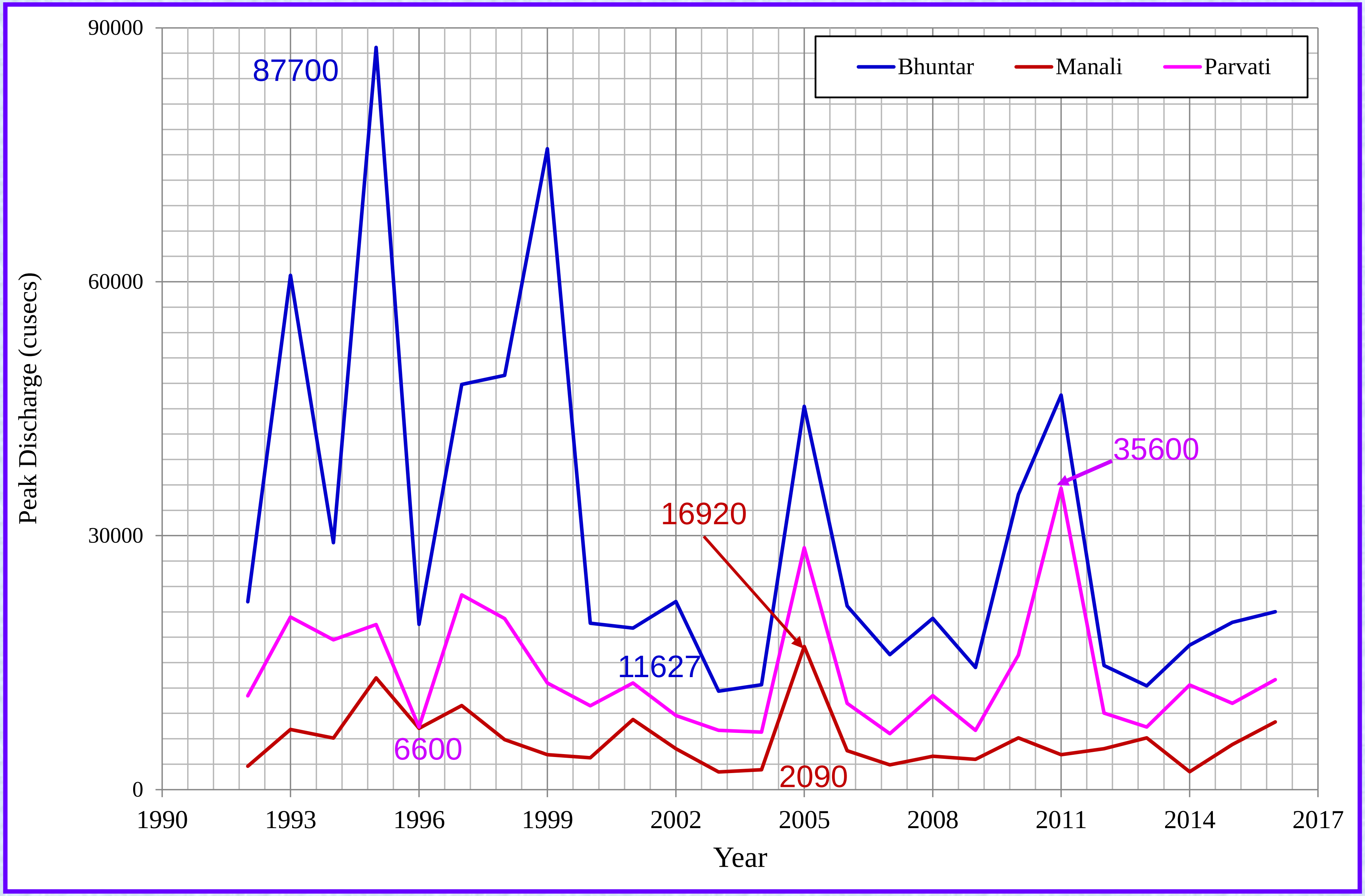
Study Area and Data Used



Index map of the study area

- Beas river basin extends between latitudes 31° 43′ to 32° 25′ N and longitudes 76° 50′ to 78° E and covers a catchment area of 3133 km^2.
- Beas is an important tributary of the Indus river, which takes off from the Rohtang Pass in the Himalayan mountain range in Himachal Pradesh (India) at an elevation of 3900 m.
- The river flows in nearly north-south direction up to Larji and takes nearly a right angle turn from which flows towards west up to the Bhuntar.

Descriptive statistics **(Stream flow data: 1992 to 2016)**	Observed			Log-transformed		
	Bhuntar	Manali	Parvati	Bhuntar	Manali	Parvati
Average (cusecs)	30426.1	5845.8	13755.0	10.144	8.532	9.414
Std. Deviation (cusecs)	20601.6	3470.5	7357.4	0.586	0.534	0.474
Coefficient of skewness	1.487	1.741	1.481	0.673	0.268	0.574
Coefficient of kurtosis	1.570	3.630	2.106	-0.607	-0.047	-0.463
Minimum (cusecs)	11627	2090	6600	9.361	7.645	8.795
Maximum (cusecs)	87700	16920	35600	11.382	9.736	10.480

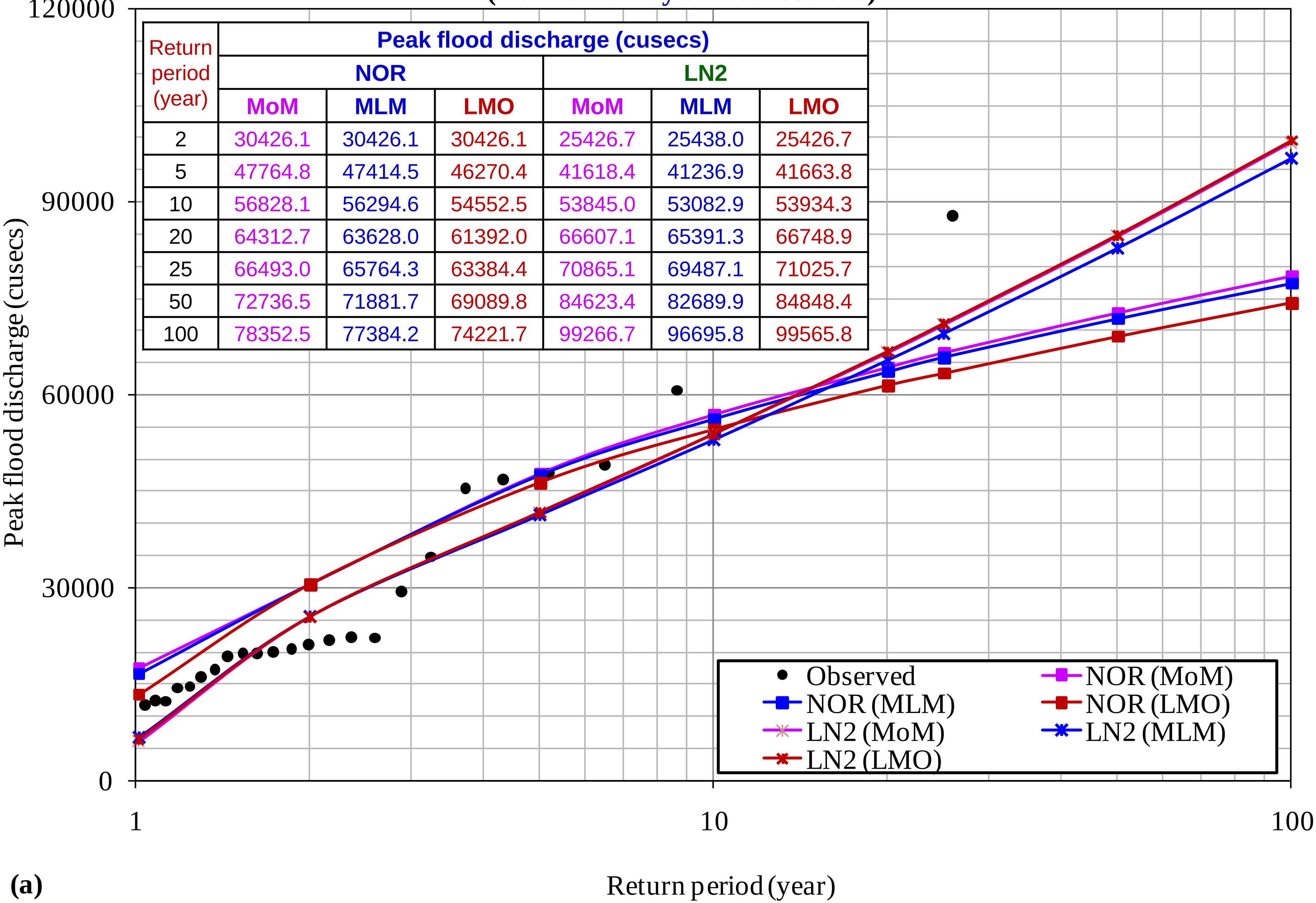


90000
60000
30000
0
87700
Bhuntar
Manali
Parvati
35600
16920
11627
6600
2090
Peak Discharge (cusecs)
1990
1993
1996
1999
2002
2005
2008
2011
2014
2017
Year

Results and Discussion

Bhuntar

(Normal Family of Distributions)

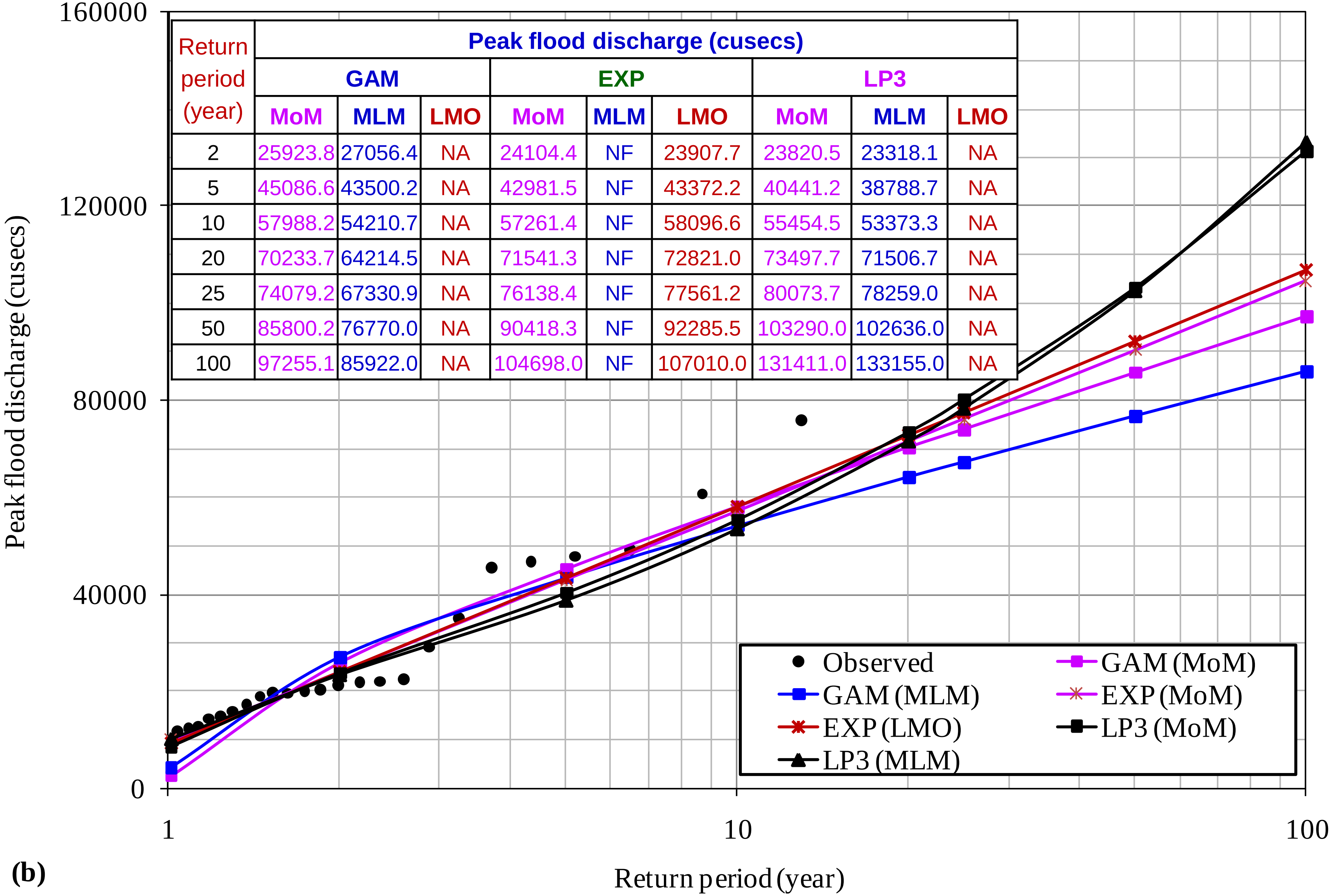


Return period (year)	Peak flood discharge (cusecs)					
	NOR			LN2		
	MoM	MLM	LMO	MoM	MLM	LMO
2	30426.1	30426.1	30426.1	25426.7	25438.0	25426.7
5	47764.8	47414.5	46270.4	41618.4	41236.9	41663.8
10	56828.1	56294.6	54552.5	53845.0	53082.9	53934.3
20	64312.7	63628.0	61392.0	66607.1	65391.3	66748.9
25	66493.0	65764.3	63384.4	70865.1	69487.1	71025.7
50	72736.5	71881.7	69089.8	84623.4	82689.9	84848.4
100	78352.5	77384.2	74221.7	99266.7	96695.8	99565.8

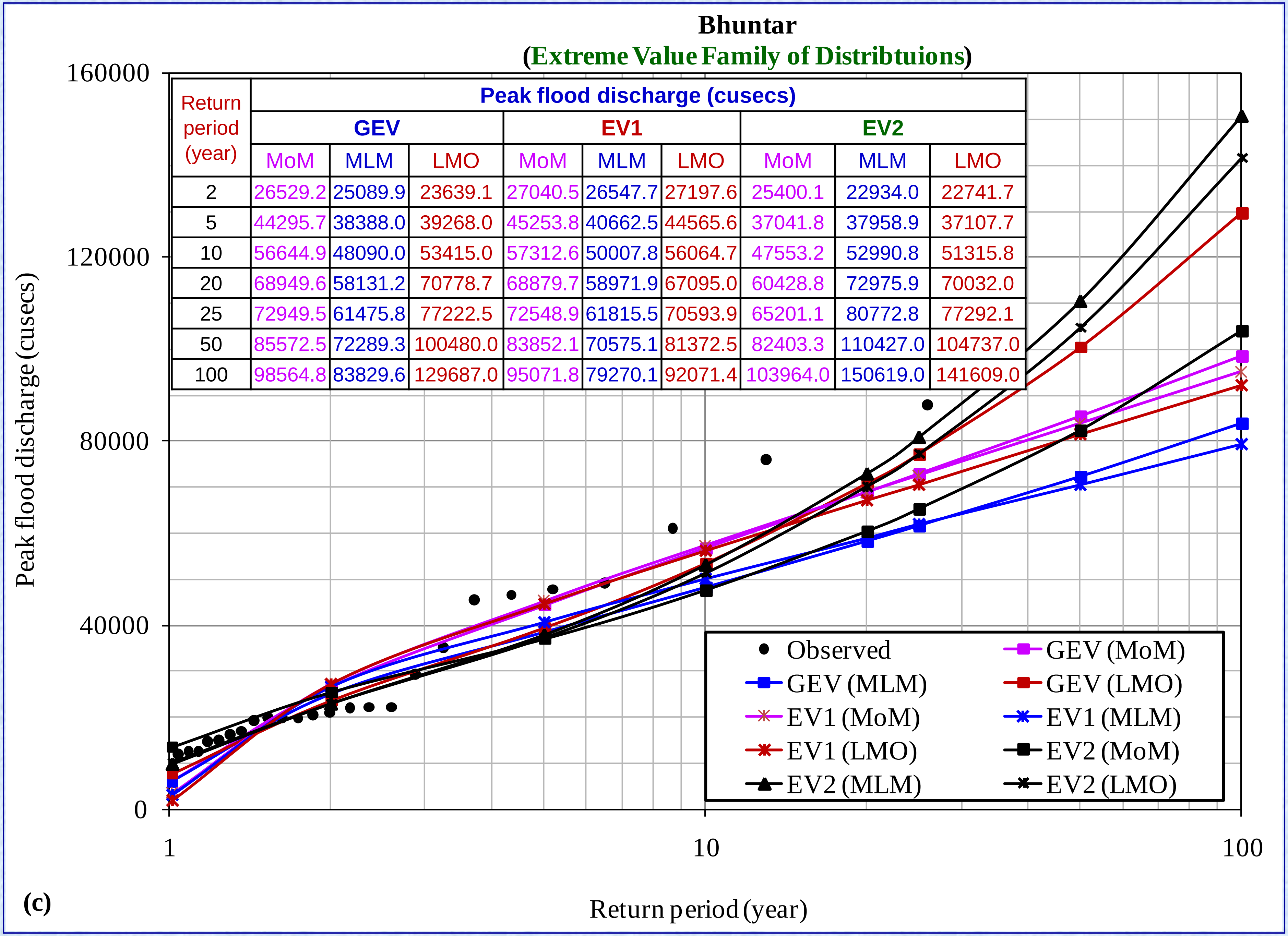
(a)

Bhuntar
(Gamma Family of Distributions)

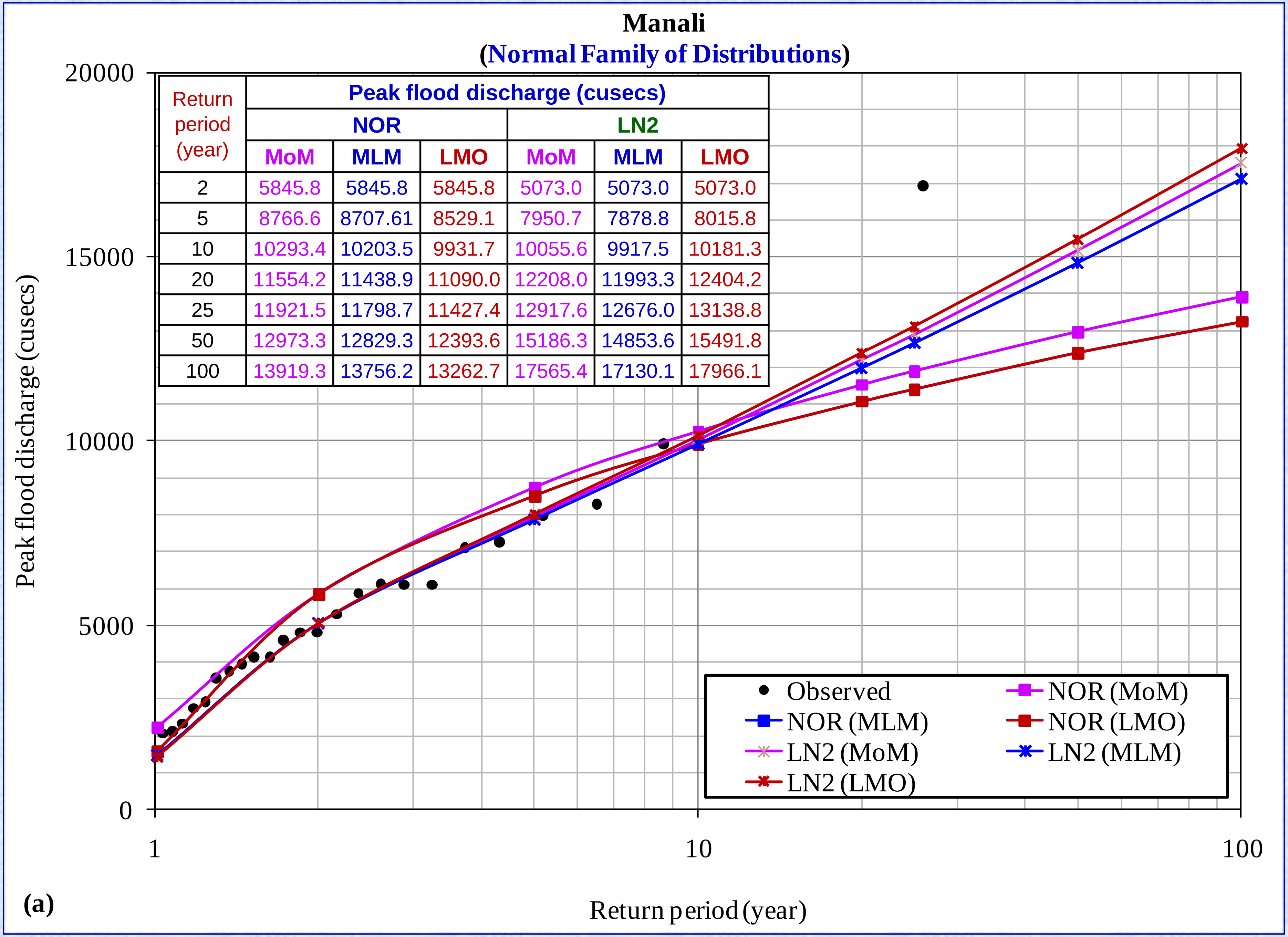
Return period (year)	Peak flood discharge (cusecs)								
	GAM			EXP			LP3		
	MoM	MLM	LMO	MoM	MLM	LMO	MoM	MLM	LMO
2	25923.8	27056.4	NA	24104.4	NF	23907.7	23820.5	23318.1	NA
5	45086.6	43500.2	NA	42981.5	NF	43372.2	40441.2	38788.7	NA
10	57988.2	54210.7	NA	57261.4	NF	58096.6	55454.5	53373.3	NA
20	70233.7	64214.5	NA	71541.3	NF	72821.0	73497.7	71506.7	NA
25	74079.2	67330.9	NA	76138.4	NF	77561.2	80073.7	78259.0	NA
50	85800.2	76770.0	NA	90418.3	NF	92285.5	103290.0	102636.0	NA
100	97255.1	85922.0	NA	104698.0	NF	107010.0	131411.0	133155.0	NA



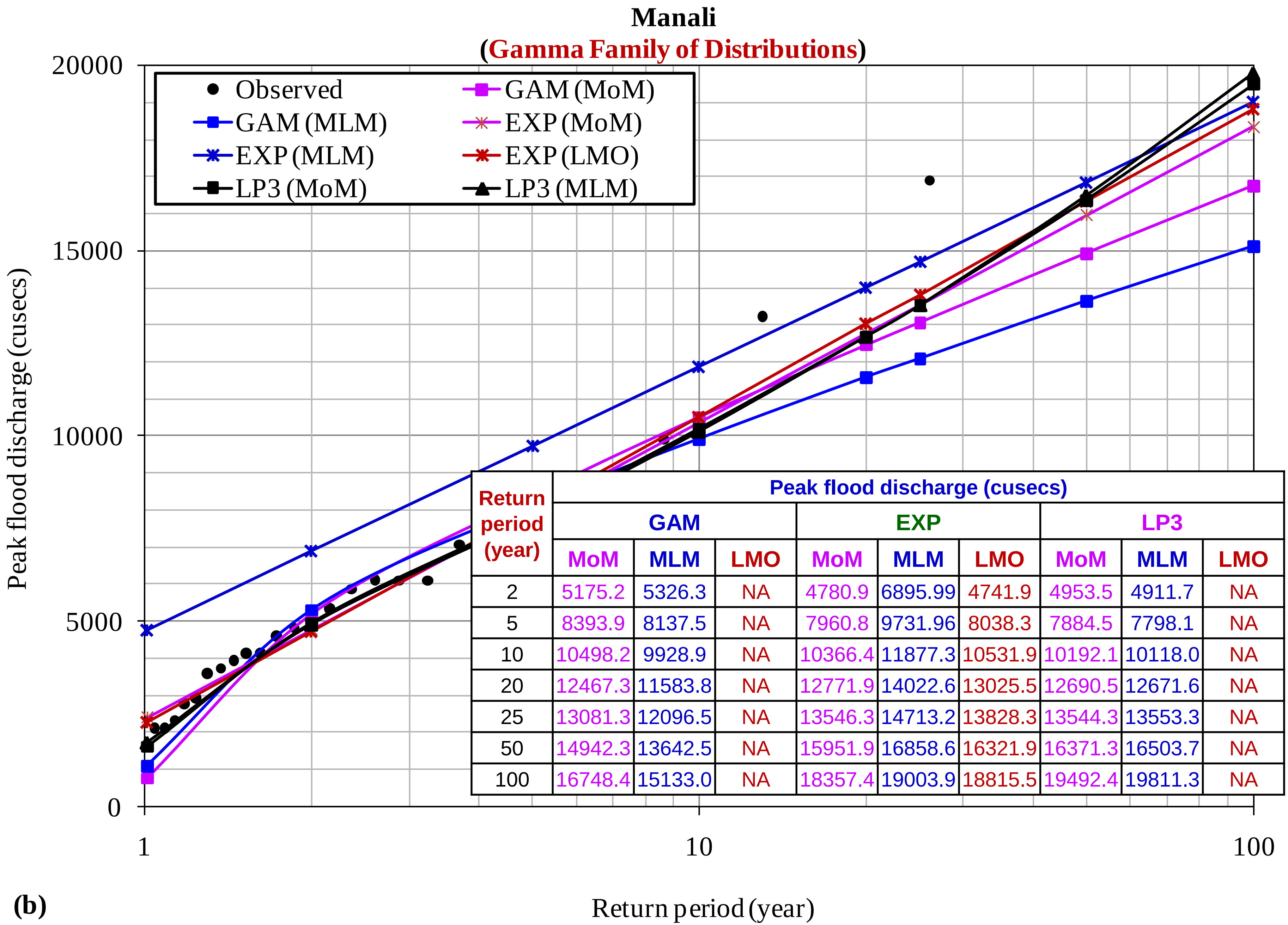
(b)



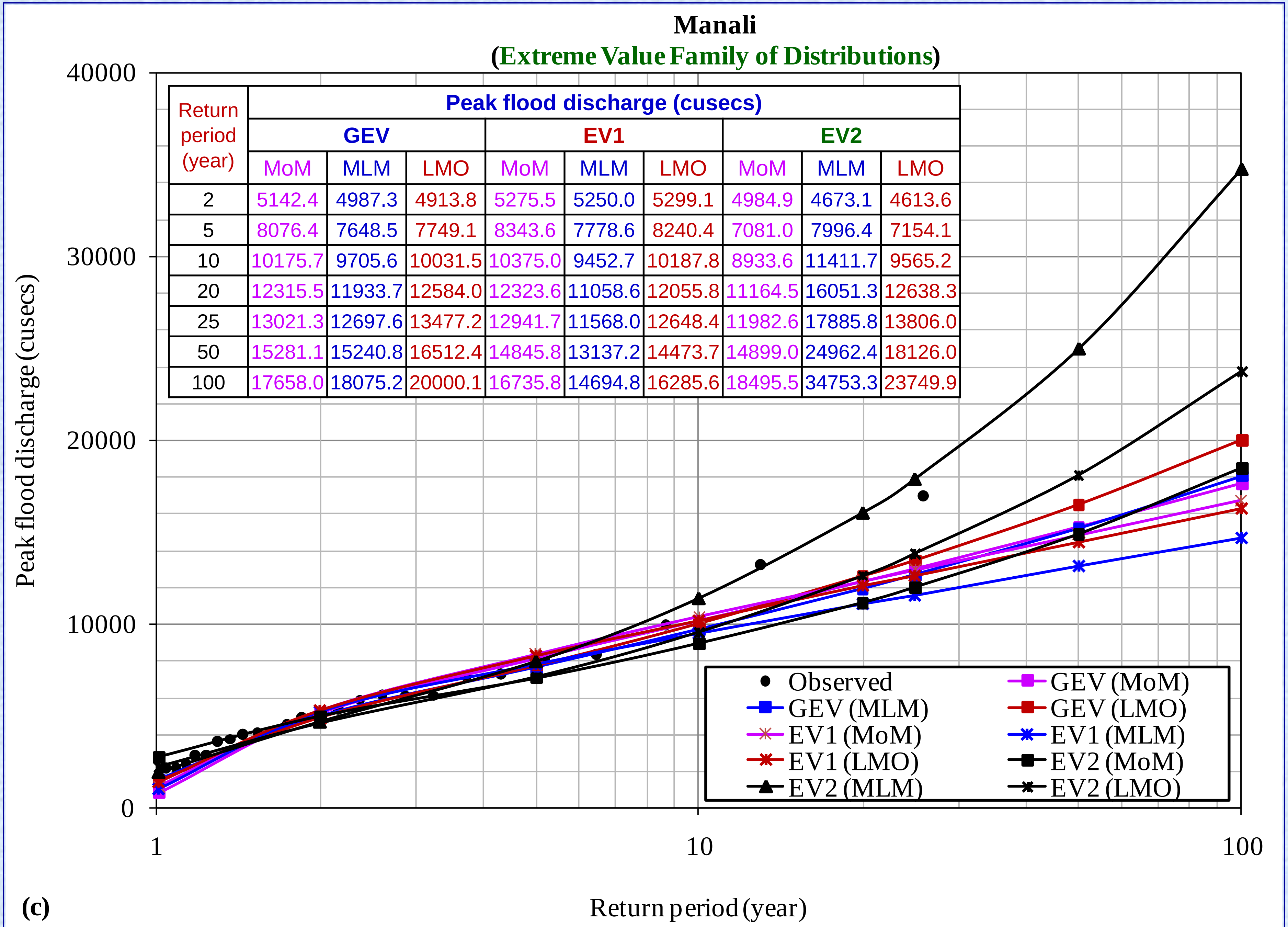
Return period (year)	Peak flood discharge (cusecs)								
	GEV			EV1			EV2		
	MoM	MLM	LMO	MoM	MLM	LMO	MoM	MLM	LMO
2	26529.2	25089.9	23639.1	27040.5	26547.7	27197.6	25400.1	22934.0	22741.7
5	44295.7	38388.0	39268.0	45253.8	40662.5	44565.6	37041.8	37958.9	37107.7
10	56644.9	48090.0	53415.0	57312.6	50007.8	56064.7	47553.2	52990.8	51315.8
20	68949.6	58131.2	70778.7	68879.7	58971.9	67095.0	60428.8	72975.9	70032.0
25	72949.5	61475.8	77222.5	72548.9	61815.5	70593.9	65201.1	80772.8	77292.1
50	85572.5	72289.3	100480.0	83852.1	70575.1	81372.5	82403.3	110427.0	104737.0
100	98564.8	83829.6	129687.0	95071.8	79270.1	92071.4	103964.0	150619.0	141609.0



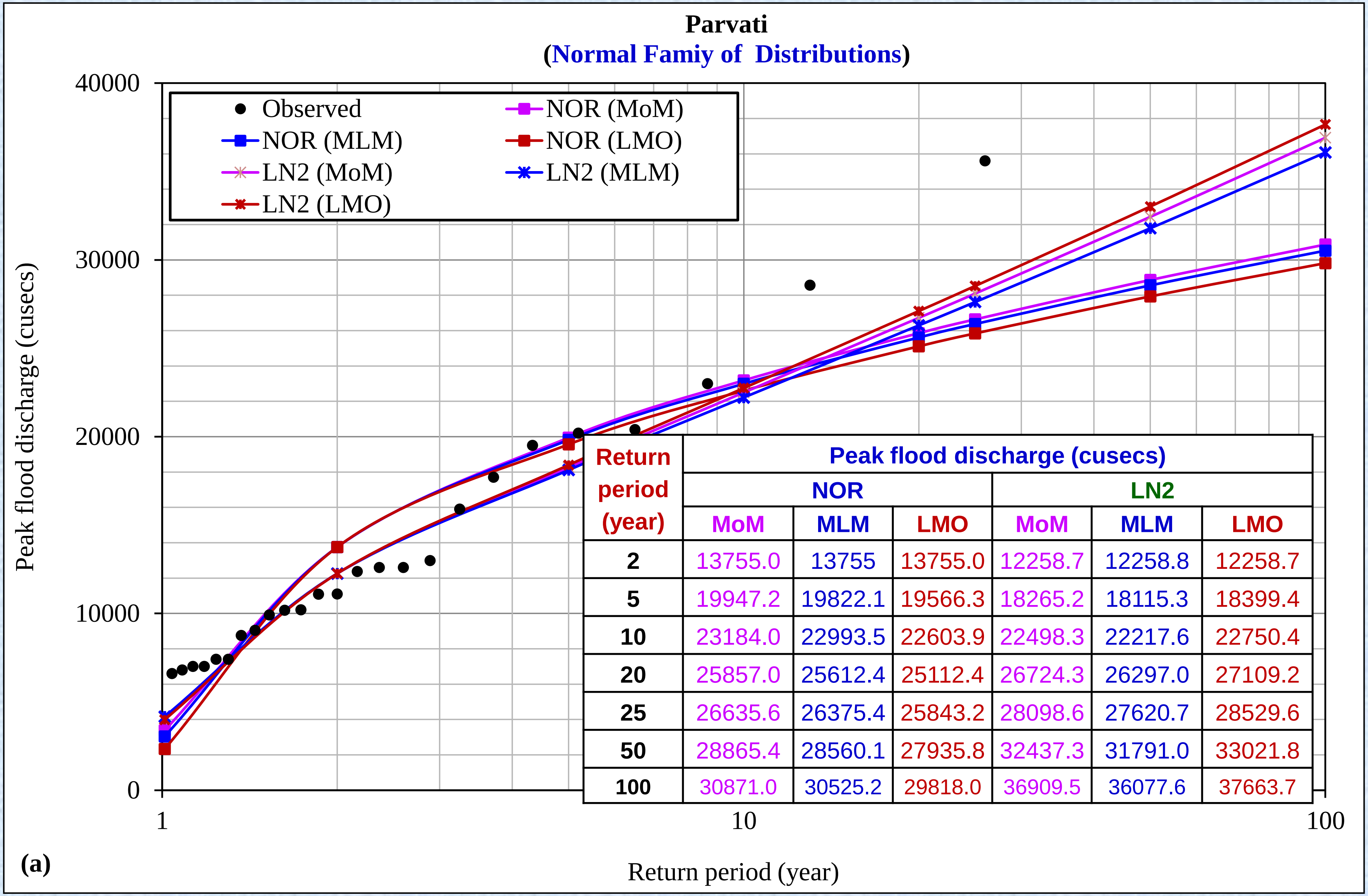
Return period (year)	Peak flood discharge (cusecs)					
	NOR			LN2		
	MoM	MLM	LMO	MoM	MLM	LMO
2	5845.8	5845.8	5845.8	5073.0	5073.0	5073.0
5	8766.6	8707.61	8529.1	7950.7	7878.8	8015.8
10	10293.4	10203.5	9931.7	10055.6	9917.5	10181.3
20	11554.2	11438.9	11090.0	12208.0	11993.3	12404.2
25	11921.5	11798.7	11427.4	12917.6	12676.0	13138.8
50	12973.3	12829.3	12393.6	15186.3	14853.6	15491.8
100	13919.3	13756.2	13262.7	17565.4	17130.1	17966.1



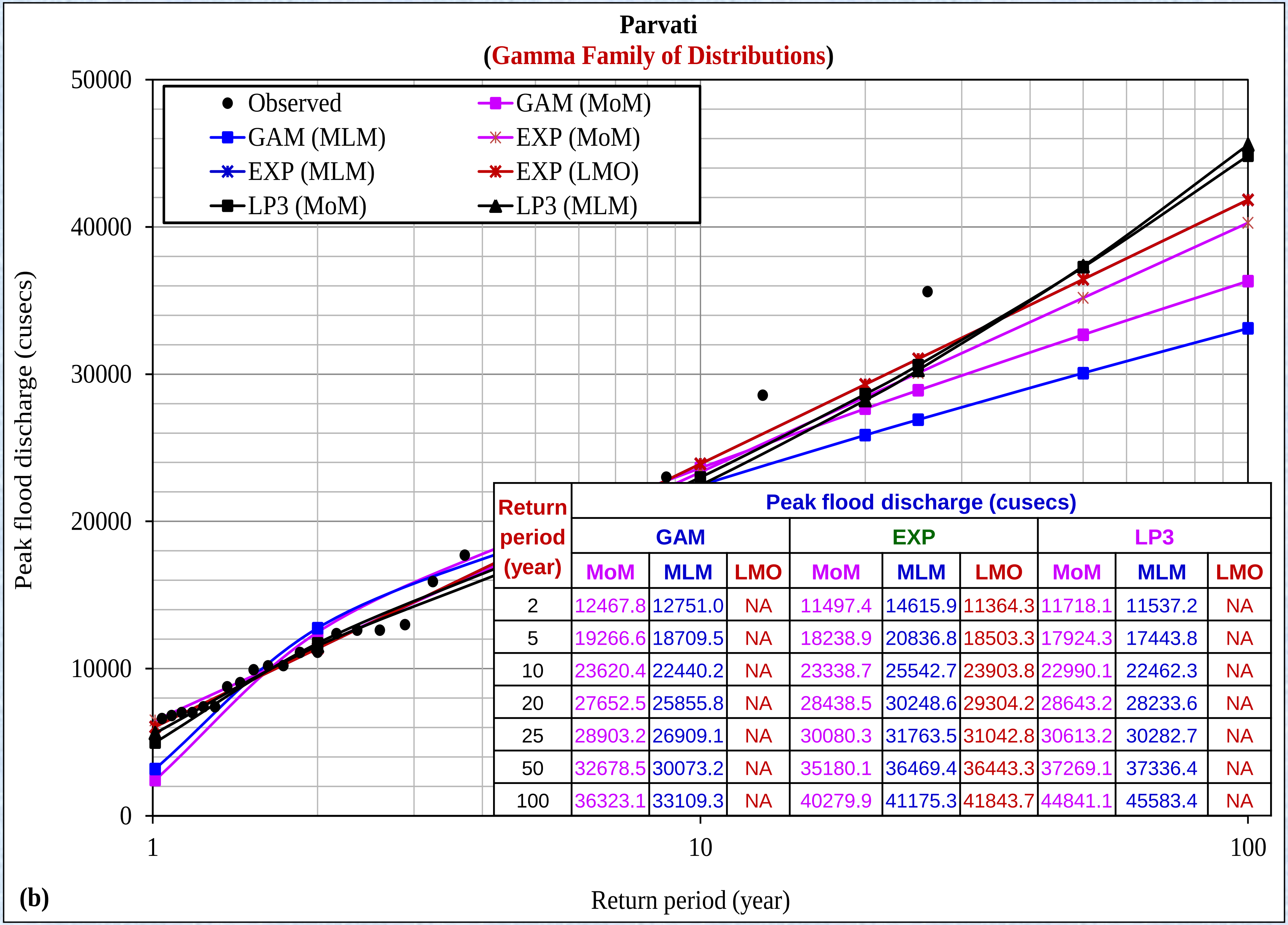
Return period (year)	Peak flood discharge (cusecs)								
	GAM			EXP			LP3		
	MoM	MLM	LMO	MoM	MLM	LMO	MoM	MLM	LMO
2	5175.2	5326.3	NA	4780.9	6895.99	4741.9	4953.5	4911.7	NA
5	8393.9	8137.5	NA	7960.8	9731.96	8038.3	7884.5	7798.1	NA
10	10498.2	9928.9	NA	10366.4	11877.3	10531.9	10192.1	10118.0	NA
20	12467.3	11583.8	NA	12771.9	14022.6	13025.5	12690.5	12671.6	NA
25	13081.3	12096.5	NA	13546.3	14713.2	13828.3	13544.3	13553.3	NA
50	14942.3	13642.5	NA	15951.9	16858.6	16321.9	16371.3	16503.7	NA
100	16748.4	15133.0	NA	18357.4	19003.9	18815.5	19492.4	19811.3	NA



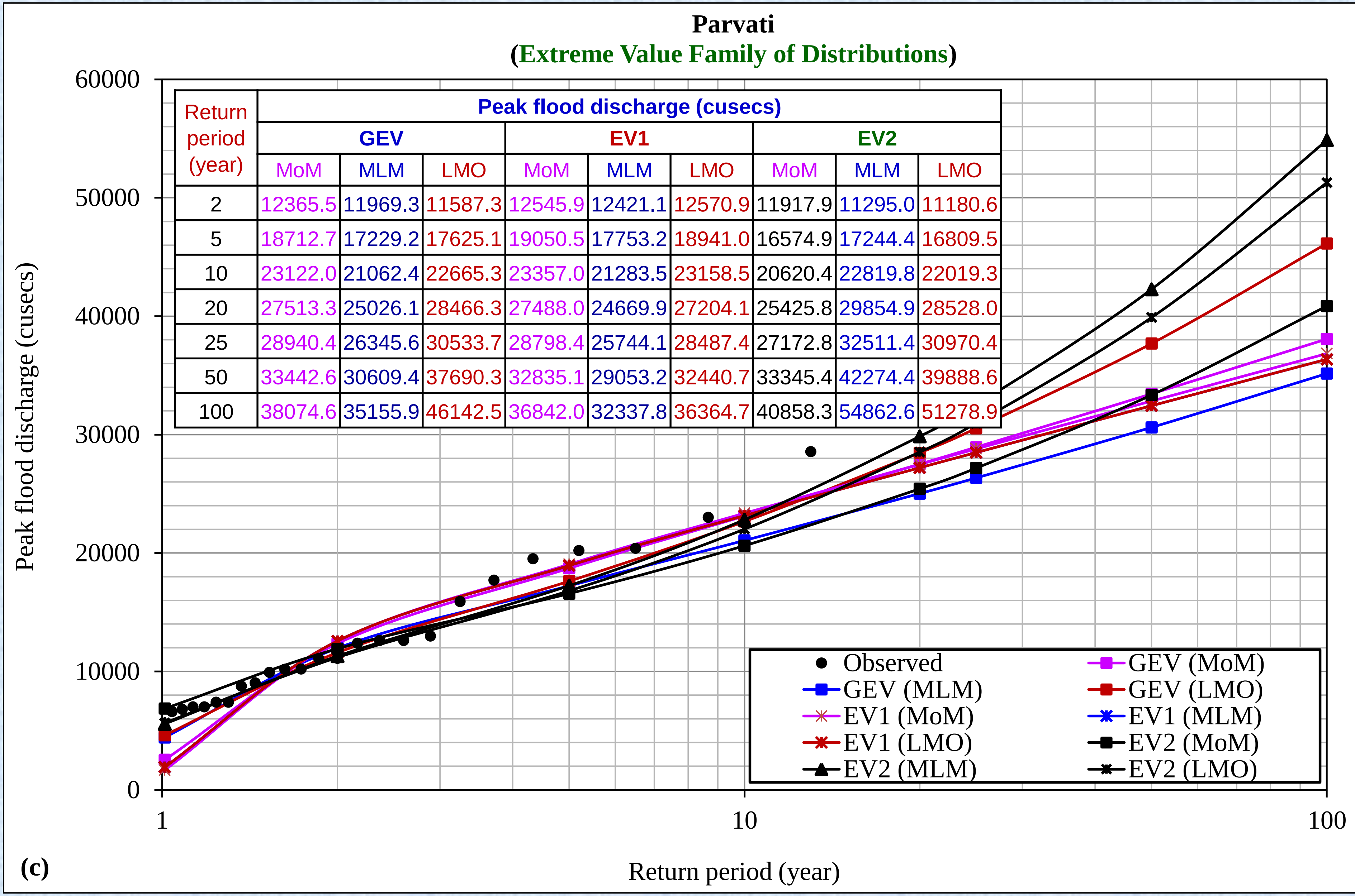
Return period (year)	Peak flood discharge (cusecs)								
	GEV			EV1			EV2		
	MoM	MLM	LMO	MoM	MLM	LMO	MoM	MLM	LMO
2	5142.4	4987.3	4913.8	5275.5	5250.0	5299.1	4984.9	4673.1	4613.6
5	8076.4	7648.5	7749.1	8343.6	7778.6	8240.4	7081.0	7996.4	7154.1
10	10175.7	9705.6	10031.5	10375.0	9452.7	10187.8	8933.6	11411.7	9565.2
20	12315.5	11933.7	12584.0	12323.6	11058.6	12055.8	11164.5	16051.3	12638.3
25	13021.3	12697.6	13477.2	12941.7	11568.0	12648.4	11982.6	17885.8	13806.0
50	15281.1	15240.8	16512.4	14845.8	13137.2	14473.7	14899.0	24962.4	18126.0
100	17658.0	18075.2	20000.1	16735.8	14694.8	16285.6	18495.5	34753.3	23749.9



Return period (year)	Peak flood discharge (cusecs)					
	NOR			LN2		
	MoM	MLM	LMO	MoM	MLM	LMO
2	13755.0	13755	13755.0	12258.7	12258.8	12258.7
5	19947.2	19822.1	19566.3	18265.2	18115.3	18399.4
10	23184.0	22993.5	22603.9	22498.3	22217.6	22750.4
20	25857.0	25612.4	25112.4	26724.3	26297.0	27109.2
25	26635.6	26375.4	25843.2	28098.6	27620.7	28529.6
50	28865.4	28560.1	27935.8	32437.3	31791.0	33021.8
100	30871.0	30525.2	29818.0	36909.5	36077.6	37663.7



Return period (year)	Peak flood discharge (cusecs)								
	GAM			EXP			LP3		
	MoM	MLM	LMO	MoM	MLM	LMO	MoM	MLM	LMO
2	12467.8	12751.0	NA	11497.4	14615.9	11364.3	11718.1	11537.2	NA
5	19266.6	18709.5	NA	18238.9	20836.8	18503.3	17924.3	17443.8	NA
10	23620.4	22440.2	NA	23338.7	25542.7	23903.8	22990.1	22462.3	NA
20	27652.5	25855.8	NA	28438.5	30248.6	29304.2	28643.2	28233.6	NA
25	28903.2	26909.1	NA	30080.3	31763.5	31042.8	30613.2	30282.7	NA
50	32678.5	30073.2	NA	35180.1	36469.4	36443.3	37269.1	37336.4	NA
100	36323.1	33109.3	NA	40279.9	41175.3	41843.7	44841.1	45583.4	NA



Return period (year)	Peak flood discharge (cusecs)								
	GEV			EV1			EV2		
	MoM	MLM	LMO	MoM	MLM	LMO	MoM	MLM	LMO
2	12365.5	11969.3	11587.3	12545.9	12421.1	12570.9	11917.9	11295.0	11180.6
5	18712.7	17229.2	17625.1	19050.5	17753.2	18941.0	16574.9	17244.4	16809.5
10	23122.0	21062.4	22665.3	23357.0	21283.5	23158.5	20620.4	22819.8	22019.3
20	27513.3	25026.1	28466.3	27488.0	24669.9	27204.1	25425.8	29854.9	28528.0
25	28940.4	26345.6	30533.7	28798.4	25744.1	28487.4	27172.8	32511.4	30970.4
50	33442.6	30609.4	37690.3	32835.1	29053.2	32440.7	33345.4	42274.4	39888.6
100	38074.6	35155.9	46142.5	36842.0	32337.8	36364.7	40858.3	54862.6	51278.9

Evaluation of Results using GoF Tests

GoF tests	NOR			LN2		
	MoM	MLM	LMO	MoM	MLM	LMO
χ^2 test						
Bhuntar	17.2	15.2	15.2	6.0	6.0	6.0
Manali	2.8	2.8	2.8	1.2	1.2	1.2
Parvati	2.0	2.0	2.0	2.4	2.4	2.4
KS test						
Bhuntar	0.270	0.277	0.284	0.196	0.196	0.198
Manali	0.161	0.159	0.158	0.052	0.052	0.053
Parvati	0.196	0.198	0.199	0.100	0.100	0.101

Normal Family of Distributions

Distribution/ method is not acceptable for fitting

Gamma Family of Distributions

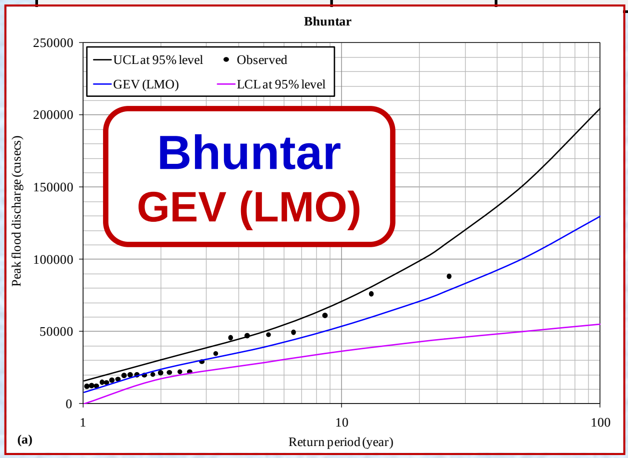
GoF tests	GAM			EXP			LP3		
	MoM	MLM	LMO	MoM	MLM	LMO	MoM	MLM	LMO
χ^2 test									
Bhuntar	10.0	10.0	10.0	6.8	6.8	5.2	3.6	3.6	3.6
Manali	1.6	1.6	1.6	0.4	0.4	2.0	0.4	0.4	0.4
Parvati	2.4	2.4	2.4	4.8	4.8	4.8	2.4	2.4	2.4
KS test									
Bhuntar	0.198	0.198	0.199	0.163	0.161	0.157	0.164	0.168	0.164
Manali	0.083	0.083	0.085	0.060	0.074	0.088	0.049	0.052	0.049
Parvati	0.125	0.125	0.128	0.103	0.085	0.062	0.095	0.098	0.095

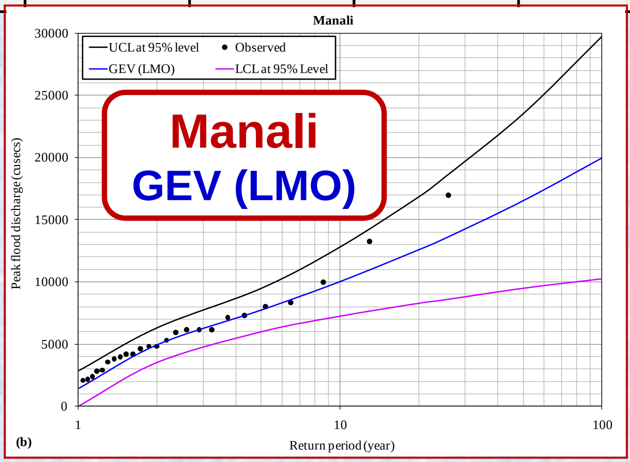
GoF tests	GEV			EV1			EV2		
	MoM	MLM	LMO	MoM	MLM	LMO	MoM	MLM	LMO
χ^2 test									
Bhuntar	20.8	8.0	3.6	20.8	6.2	20.8	11.2	5.2	2.8
Manali	0.4	0.4	0.4	1.2	1.2	0.4	2.8	2.0	1.2
Parvati	2.4	2.4	4.4	2.4	2.4	2.4	6.8	4.8	2.0
KS test									
Bhuntar	0.218	0.212	0.160	0.223	0.196	0.232	0.260	0.157	0.138
Manali	0.067	0.050	0.058	0.090	0.052	0.089	0.173	0.088	0.115
Parvati	0.116	0.098	0.087	0.128	0.100	0.129	0.202	0.062	0.118

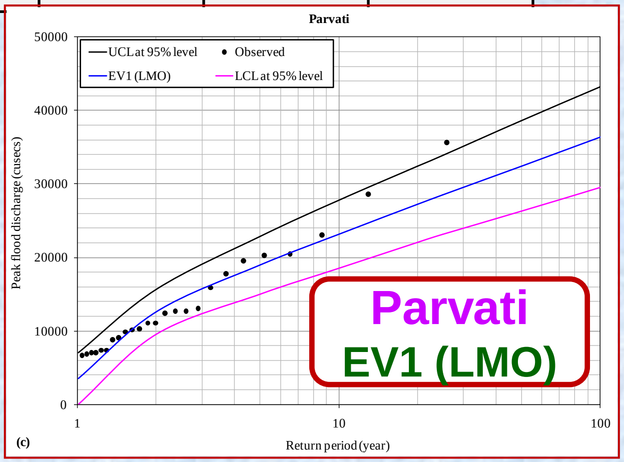
Extreme Value Family of Distributions

Evaluation of EVA Results using D-index

Probability distribution	D-index values								
	Bhuntar			Manali			Parvati		
	MoM	MLM	LMO	MoM	MLM	LMO	MoM	MLM	LMO
NOR	1.527	1.571	1.787	1.807	1.799	1.861	1.078	1.141	1.242
LN2	1.648	2.072	1.853	1.188	1.303	1.241	1.044	1.380	1.210
GAM	1.279	1.864	NA	1.366	1.540	NA	0.914	1.290	NA
EXP	1.365	NF	1.219	1.044	1.759	1.017	0.980	0.715	0.782
LP3	1.567	1.928	NA	1.055	1.051	NA	1.050	1.254	NA
GEV	1.415	2.968	1.922	1.257	1.330	1.084	1.014	1.842	1.166
EV1	1.358	2.618	1.520	1.365	1.642	1.407	0.969	1.755	1.026
EV2	2.983	1.933	2.258	1.968	0.589	1.346	1.969	1.051	1.389







Plots of estimated peak flood discharge by selected distribution with 95% confidence limits and observed annual maximum discharge

Conclusions

Intercomparison on statistical distributions for estimation of peak flood discharge (PFD) was carried out through quantitative assessment (viz., Chi-square, Kolmogorov-Smirnov and D-index) & quantitative assessment using fitted curves of the estimated PFDs.

a) For the return periods from 25-year and above, estimated PFD by EV2 (MLM) are higher than those values of GEV, EV1 and EV2 distributions for all three sites.

b) For Bhuntar, χ^2 test results indicated that the EXP (LMO), LP3 (using MoM, MLM and LMO), GEV (LMO) and EV2 (MLM and LMO) distributions are acceptable for flood estimation.

c) KS test results showed that the distributions other than NOR (using MoM, MLM and LMO) is acceptable for estimation of PFD for Bhuntar.

d) $\chi^2$and KS tests results confirmed the applicability of all eight distributions adopted in FFA for Manali and Parvati while MoM, MLM and LMO are applied in determining the parameters of the distributions.

e) Quantitative and qualitative assessments indicated that LMO provides better results when compared with MoM and MLM.

f) Study suggested that the PFD values for different return periods given by GEV (LMO) for Bhuntar and Manali whereas EV1 (LMO) for Parvati could be used for design purposes.

Recommendation

- Study suggested that the estimated peak flood discharges for different return periods obtained from
  - GEV (LMO) for Bhuntar
  - GEV (LMO) for Manali
  - EV1 (LMO) for Parvati

  could be used for planning and design of hydraulic structures such as barrages, canals, bridges, dams, embankments, reservoirs and spillways, insurance studies, planning of flood management and rescue operations.
- By considering the data length (i.e., 25 years) made available for the study, the study suggested that the PFD for return period beyond 75-year may be cautiously used due to uncertainty in the higher order return periods.

Comparison of Estimators of Weibull Distribution for Low-flow Frequency Analysis

Outline of the Presentation

- Introduction
- Objectives of the study
- Methodology
- Study area and data used
- Evaluation of parameter estimation methods of Weibull distribution adopted in low-flow frequency analysis
- Results and Discussions
- Recommendations

Introduction

- Low-flow analysis is an important aspect for water quality management, reservoir storage design, determining minimum release policy and safe surface water withdrawals.

- Low-flow is seasonal phenomenon and an integral component of flow regime of any river.

- Hydrological literature described that there are many inter-linking natural factors, which contributes to low-flow that includes direct river withdrawals for human activity and artificial afforestation in the catchment.

- Numbers of indices such as mean annual runoff, mean daily flow, median flow, Annual Minimum d-day Average Flow (AMdAF), absolute minimum flow are widely used to characterize the low-flow.

Introduction (Contd...)

- An associated, annual event based, low-flow statistic q(d,T) gives a low-flow estimate, which is defined as the AMdAF that is expected to be occurred once in T-year return period.

- Generally, the available stream flow data are insufficient to conduct an accurate analysis of an extreme low-flow event. Therefore, for improving the accuracy of estimated low-flow, number of probability distributions is applied.

- After extracting the AMdAF series from daily stream flow data series, probability distributions are adopted in Low-flow Frequency Analysis (LFA) to estimate the value of q(d,T).

Objectives of the Study

- Estimation of low-flows for different duration (d) and return period (T) using Method of Moments (MoM), Maximum Likelihood Method (MLM), L-Moments (LMO), Probability Weighted Moments (PWM) and Principle of Maximum Entropy (PME) estimators of 2-parameters Weibull (WB2) distribution.

- Checking the adequacy of fitting WB2 distribution to the AMdAF series for different values of 'd' viz., 7-, 10-, 14- and 30- days through
  - Quantitative assessment using
    - a) Goodness-of-Fit tests viz., Chi-Square and Kolmogorov-Smirnov.
    - b) Diagnostic test viz., Correlation Coefficient (CC) and Root Mean Squared Error (RMSE).
  - Qualitative assessment using the fitted curves of the estimated low-flows.

- Analyze the results and made recommendations thereof.

Methodology

Analysis of low-flow of a stream pre-supposes that:

(i) No significant withdrawals and diversions from the location points are in operation and

(ii) Flow in a river or stream to be natural.

Distribution	Probability Distribution Function (PDF)	Cumulative Distribution Function (CDF)	Quantile estimator q(d,T)
2-parameter Weibull	$f(q)=\frac{\beta}{\alpha}\left(\frac{q}{\alpha}\right)^{\beta-1} e^{-\left(\frac{q}{\alpha}\right)^{\beta}}$ $q>0, \alpha>0, \beta>0$	$F(q)=1-e^{-\left(\frac{q}{\alpha}\right)^{\beta}}$	$q(d,T)=\hat{\alpha}\left(-\ln\left(1-\left(1/T\right)\right)\right)^{\left(1/\hat{\beta}\right)}$

$\hat{\alpha}$	Estimator of scale parameter (α)			$\hat{\beta}$	Estimator of shape parameter
f(q)	PDF of q	F(q)	CDF of q	T	Return period (year)

Methodology (Contd…)

Determination of estimators of parameters of WB2

Parameter Estimation Method	Estimator of the scale parameter ($\hat{\alpha}$)	Estimator of the shape parameter ($\hat{\beta}$)
MoM	$\mu = \hat{\alpha}\Gamma\left(1+\left(1/\hat{\beta}\right)\right)$	$\sigma^2 = \hat{\alpha}^2\left[\Gamma\left(1+\left(2/\hat{\beta}\right)\right) - \Gamma^2\left(1+\left(1/\hat{\beta}\right)\right)\right]$
MLM	$\frac{\hat{\beta}}{\hat{\alpha}}\sum_{i=1}^{N}\left(\frac{q(i)}{\hat{\alpha}}\right)^{\hat{\beta}} - \frac{N\hat{\beta}}{\hat{\alpha}} = 0$	$\frac{N}{\hat{\beta}} - N\ln(\hat{\alpha}) + \sum_{i=1}^{N}\ln(q(i)) - \sum_{i=1}^{N}\left(\frac{q(i)}{\hat{\alpha}}\right)^{\hat{\beta}}\ln\left(\frac{q(i)}{\hat{\alpha}}\right) = 0$
LMO	$\lambda_1 = \hat{\alpha}\Gamma\left(1+\left(1/\hat{\beta}\right)\right)$	$\lambda_2 = \hat{\alpha}\left(1-2^{-(1/\hat{\beta})}\right)\Gamma\left(1+\left(1/\hat{\beta}\right)\right)$
	$\lambda(1) = b(0) = \frac{\sum_{i=1}^{N} q(i)}{N}$, $\lambda(2) = 2b(1) - b(0)$ with $b(1) = \frac{\sum_{i=1}^{N}(i-1)q(i)}{N(N-1)}$	
PWM	$\hat{\alpha} = \mu(0)/\Gamma\left(1+\left(1/\hat{\beta}\right)\right)$	$\hat{\beta} = \ln(2)/\left[\ln\left(\mu(0)/\mu(1)\right) - \ln(2)\right]$
	$\mu(0) = \frac{\sum_{i=1}^{N} q(i)}{N}$ and $\mu(1) = \frac{\sum_{i=1}^{N}(N-i)q(i)}{N(N-1)}$	
PME	$\hat{\alpha} = \left(\frac{1}{N}\sum_{i=1}^{N} q(i)^{\hat{\beta}}\right)^{1/\hat{\beta}}$	$\hat{\beta} = \frac{\pi^2/6}{\frac{1}{N}\left(\sum_{i=1}^{N}\ln(q(i))^2\right) - \left(\frac{1}{N}\sum_{i=1}^{N}\ln(q(i))\right)^2}$

wherein

μ	Average of the observed data	σ	Standard deviation of observed data
b(o), b(1)	First and second sample moments	$\lambda(1)$, $\lambda(2)$	First and second L-Moments (LMOs)
q(i)	Observed low-flow (q) of i^{th} sample	ln(q(i))	Logarithmic value of observed low-flow
N	Sample size	Γ	Gamma function

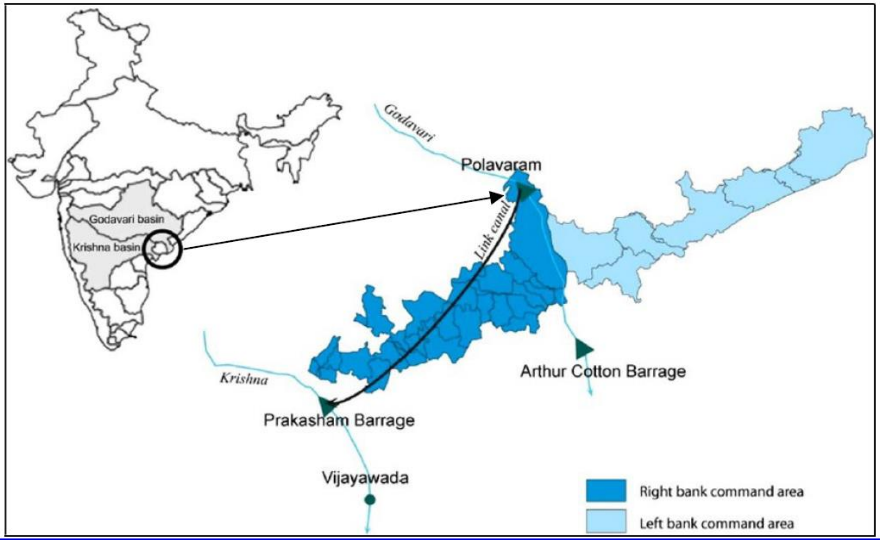
Goodness-of-Fit Test and Model Indicators Used for Evaluation of Results

Statistical Tests	Tests statistic (with description of symbols/ terms)		
GoF tests			
Chi-square (χ^2)	$\chi^2 = \sum_{j=1}^{NC} \frac{(O_j(q) - E_j(q))^2}{E_j(q)}$	$O_j(q)$	Observed frequency value of q for j^{th} class.
		$E_j(q)$	Expected frequency value of q for j^{th} class.
		NC	Number of frequency class.
Kolmogorov-Smirnov (KS)	$KS = \text{Max} \sum_{i=1}^{N} \lvert F_e(q(i)) - F_D(q(i)) \rvert$	$F_e(q(i))$	Empirical CDF of q(i) for i=1,2,3,…,N with q(1)<q(2)<….<q(N).
		$F_D(q(i))$	Derived CDF of q(i) for i=1,2,3,..., N
		N	Sample size
Diagnostic test			
Correlation Coefficient (CC)	$\frac{\sum_{i=1}^{N}(q(i)-\bar{q})(q^*(i)-\overline{q^*})}{\sqrt{\sum_{i=1}^{N}(q(i)-\bar{q})^2 \sum_{i=1}^{N}(q^*(i)-\overline{q^*})^2}}$	$q(i)$	Observed low-flow for i^{th} sample
		$q^*(i)$	Estimated low-flow for i^{th} sample
		$\bar{q}$	Average of observed low-flows
		$\overline{q^*}$	Average of estimated low-flows
Root Mean Squared Error (RMSE)	$\left(\frac{1}{N}\sum_{i=1}^{N}(q(i)-q^*(i))^2\right)^{1/2}$		

Test criteria for GoF tests: If the computed values of GoF tests statistic given by the distribution are less than that of its theoretical value at the desired significance level then the selected method of WB2 is acceptable for LFA.

Selection criteria of Diagnostic test: The method with high CC (say, CC>0.9) and minimum RMSE is identified as a better-suited for estimation of low-flow.

Study Area and Data Used



Location map of river Godavari at Polavaram

- Main tributaries of the river are the Manjira, the Pranhita, the Indravati and the Sabari.
- Out of the total average annual flow of the river, nearly 40% is contributed by the Pranhita, 20% by the Indravati, 10% by the Sabari and the rest by the other tributaries and the Godavari itself.
- The daily stream flow data observed at Polavaram gauging site for the period 1972 to 2012 is used.

- The river Godavari rises in the Western Ghats at Triambak near Nasik about 113 km (kilometer) North-East of Bombay and only 80 km from the Arabian Sea.

- After descending the Western Ghats, it takes a South-Easterly course across the southern part of Indian Peninsula and flows through 1230 km and falls into the Bay of Bengal about 80 km east of Rajahmundry.

- The Polavaram gauging site is located in the Godavari river basin and lies at 17° 15′ 07″ N latitude and 81° 39′ 23″ E longitude.

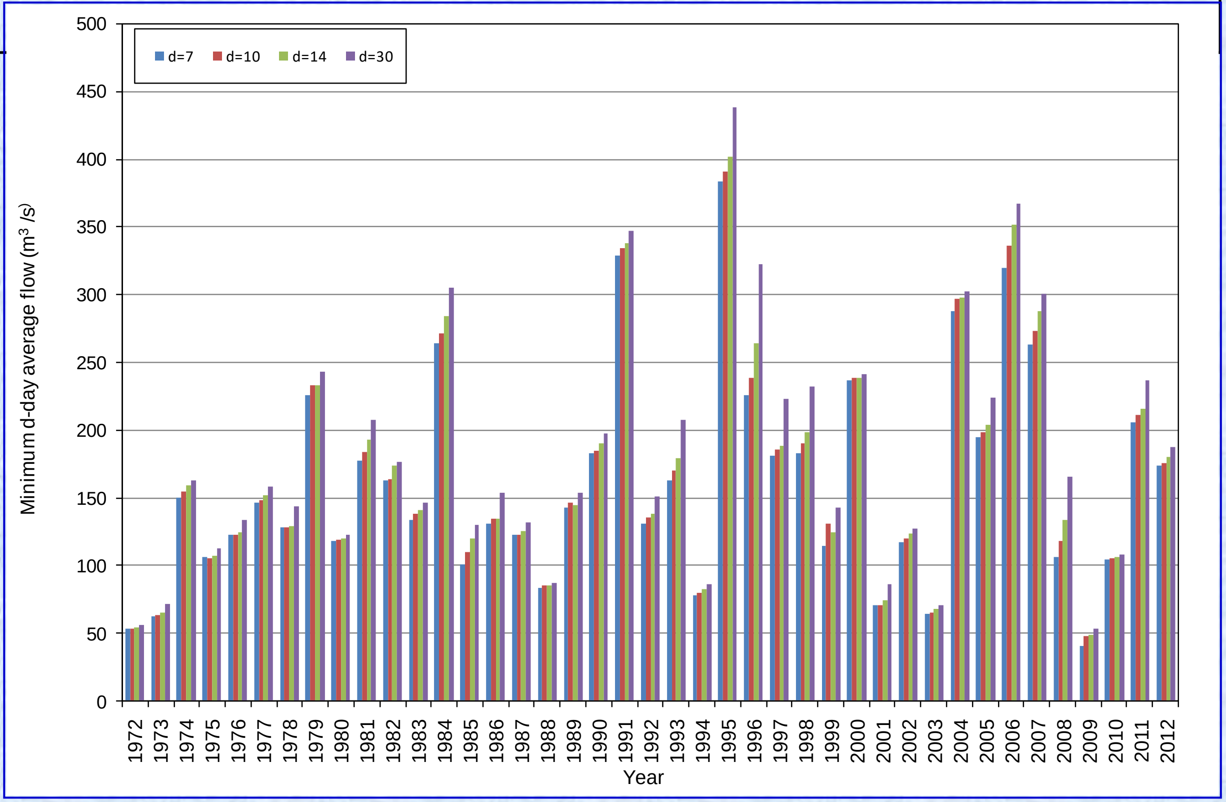
- Total catchment area of Godavari river is 312812 km^2. The catchment area of the Polavaram site is 307800 km^2.

Descriptive Statistics of Observed AMdAF

Descriptive statistics	AMdAF for different values of 'd'			
	d=7	d=10	d=14	d=30
Average (m^3/s)	160.3	165.1	169.9	182.9
SD (m^3/s)	79.2	81.4	84.2	90.2
CS	0.926	0.925	0.912	0.865
CK	0.596	0.536	0.457	0.426

SD: Standard Deviation; CS: Coefficient of Skewness; CK: Coefficient of Kurtosis

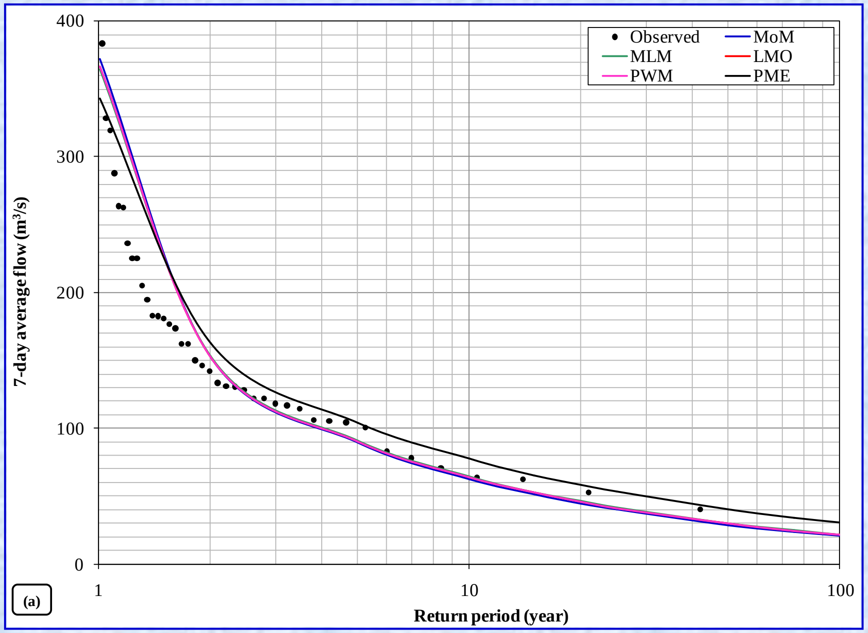
Time series plots of the observed annual minimum d-day average flows (for d=7, 10-, 14- and 30-days)

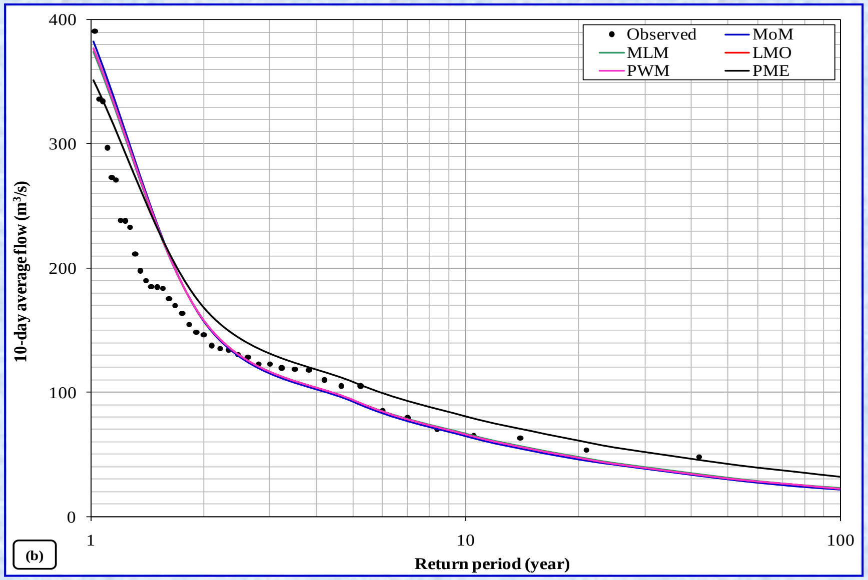


Estimated low-flows for different return periods using WB2 distribution

Return period T(year)	q(7,T) (m^3/s)				
	MoM	MLM	LMO	PWM	PME
1.01	372.0	364.6	367.0	367.0	342.8
2	152.3	153.1	152.8	152.8	162.7
5	89.3	91.1	90.5	90.5	104.2
10	62.7	64.6	64.0	64.0	77.6
15	51.4	53.2	52.6	52.6	65.7
20	44.7	46.5	45.9	45.9	58.5
25	40.1	41.9	41.3	41.3	53.5
50	28.8	30.3	29.8	29.8	40.5
75	23.8	25.2	24.7	24.7	34.5
100	20.7	22.0	21.6	21.6	30.8

Return period T(year)	q(10,T) (m^3/s)				
	MoM	MLM	LMO	PWM	PME
1.01	382.4	374.3	377.0	377.0	351.5
2	156.9	157.7	157.4	157.4	168.1
5	92.1	94.0	93.4	93.4	108.1
10	64.7	66.8	66.1	66.1	80.7
15	53.0	55.1	54.4	54.4	68.5
20	46.1	48.1	47.5	47.5	61.0
25	41.4	43.3	42.7	42.7	55.8
50	29.8	31.5	30.9	30.9	42.5
75	24.6	26.1	25.6	25.6	36.2
100	21.4	22.9	22.4	22.4	32.4

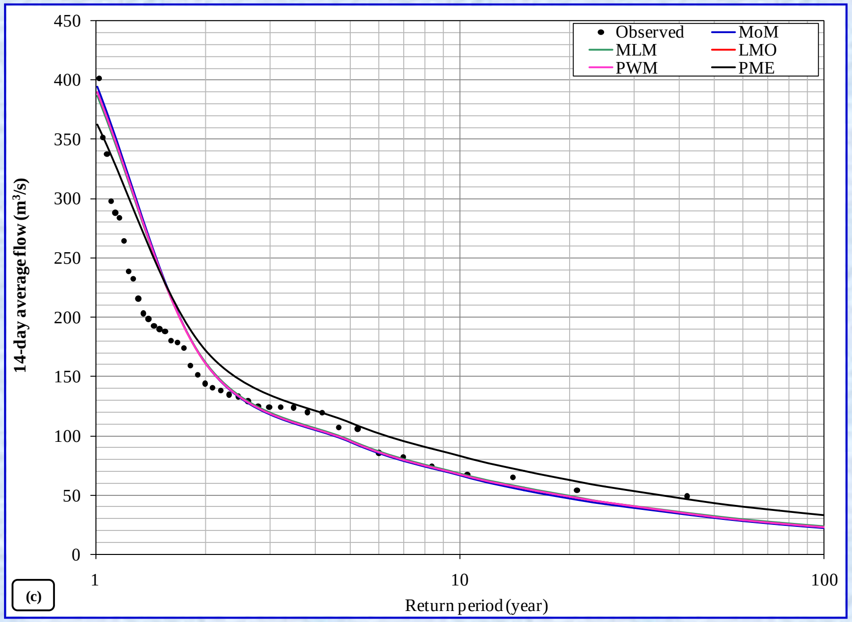


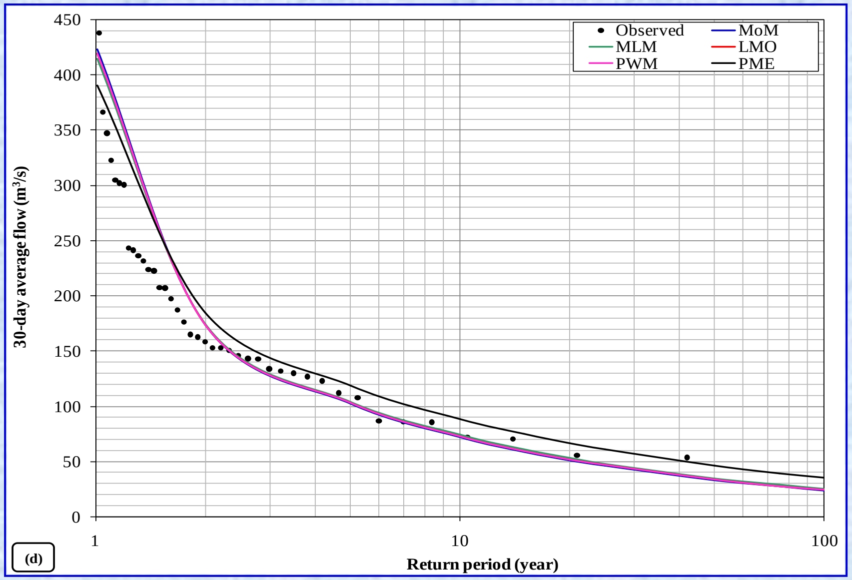


Estimated low-flows for different return periods using WB2 distribution

Return period T(year)	q(14,T) (m^3/s)				
	MoM	MLM	LMO	PWM	PME
1.01	394.8	386.4	389.6	389.6	362.8
2	161.3	162.2	161.9	161.9	172.8
5	94.5	96.5	95.7	95.7	111.0
10	66.3	68.4	67.6	67.6	82.7
15	54.3	56.4	55.6	55.6	70.1
20	47.2	49.2	48.4	48.4	62.4
25	42.4	44.3	43.6	43.6	57.1
50	30.4	32.1	31.5	31.5	43.4
75	25.1	26.6	26.0	26.0	37.0
100	21.9	23.3	22.8	22.8	33.0

Return period T(year)	q(30,T) (m^3/s)				
	MoM	MLM	LMO	PWM	PME
1.01	423.5	414.9	419.8	419.8	390.9
2	173.9	174.7	174.2	174.2	185.3
5	102.1	104.2	103.0	103.0	118.6
10	71.8	74.0	72.7	72.7	88.3
15	58.8	61.0	59.8	59.8	74.7
20	51.2	53.3	52.1	52.1	66.5
25	46.0	48.0	46.8	46.8	60.8
50	33.1	34.8	33.8	33.8	46.1
75	27.3	28.9	28.0	28.0	39.2
100	23.8	25.3	24.5	24.5	35.0





Discussions on LFA Results and LFC s

- **Estimated low-flow (q(d,T)) using five methods of WB2 Distribution**
  - Low-flow estimates obtained from MoM are lower than the corresponding values given by MLM, LMO, PWM and PME for return periods from 5-year to 100-year.
  - There is no difference between the low-flow estimates obtained from LMO and PWM of WB2 though the procedures involved in determining the parameters using LMO and PWM are different.

- **Low-Flow Frequency Curves (LFCs)**
  - About 25% of the observed AMdAF corresponding to return period below 2-years are gathered below the fitted lines of the low-flow estimates using WB2.
  - This could be due to the more pronouncements of difference in observed and estimates for return period below 2-years as one of the drawback of probability plot.
  - Observed low-flows are nearer to the estimated low-flows given by LMO (or PWM) of WB2.

Analysis of Results Based on GoF Test

AMdAF series	Computed values of GoF tests statistic using									
	χ^2					KS				
	MoM	MLM	LMO	PWM	PME	MoM	MLM	LMO	PWM	PME
d=7	1.585	1.585	1.585	1.585	1.723	0.075	0.075	0.075	0.075	0.080
d=10	2.171	2.171	1.000	1.000	1.312	0.069	0.069	0.073	0.073	0.076
d=14	1.585	1.585	1.585	1.585	1.723	0.081	0.081	0.084	0.084	0.087
d=30	1.293	1.293	1.293	1.293	1.415	0.086	0.086	0.088	0.088	0.091

Inference:

The computed values by different parameter estimation methods of WB2 are less than its theoretical value at 5% (viz., 7.820 for χ^2 and 0.199 for KS), and at this level, all five methods of WB2 are acceptable for LFA.

Analysis of Results Based on Diagnostic Test

AMdAF series	CC					RMSE (m^3/s)				
	MoM	MLM	LMO	PWM	PME	MoM	MLM	LMO	PWM	PME
d=7	0.987	0.986	0.986	0.986	0.980	13.0	14.0	13.7	13.7	19.9
d=10	0.986	0.985	0.985	0.985	0.979	13.9	14.9	14.6	14.6	21.2
d=14	0.986	0.985	0.986	0.986	0.979	14.3	15.4	14.9	14.9	21.8
d=30	0.988	0.987	0.988	0.988	0.982	14.4	15.6	14.9	14.9	22.1

Inferences:

- MoM gave high CC and minimum RMSE when compared with the corresponding values of MLM, LMO, PWM and PME for the series of AMdAF for different values of ‘d’ such as 7-, 10-, 14-, and 30-days.
- MoM estimates are often less accurate than those obtained by other parameter estimation procedures such as MLM, LMO, PWM and PME.
- After eliminating the diagnostic test results of WB2 (using MoM) related to the AMdAF series for different values of ‘d’ such as 7-, 10-, 14-, and 30-days, it is noted that LMO and PWM gave good CC and minimum RMSE when compared with those values of MLM and PME.
- In view of the above, it is suggested that WB2 (using LMO or PWM) could be considered as an appropriate parameter estimation method of WB2 for estimation of low-flow for river Godavari at Polavaram site.

Conclusions

- Estimated low-flows by MoM are lower than those values of MLM, LMO, PWM and PME for return periods from 5-year to 100-year.
- Observed low-flows are nearer to the estimated low-flows given by LMO (or PWM) of WB2.
- Estimated low-flows using LMO and PWM are found to be same though the procedures involved in determining the parameters of WB2 using LMO and PWM are different.
- GoF and diagnostic tests results confirmed that LMO (or PWM) of WB2 could be considered as an appropriate method for estimation of low-flows.
- RMSE on the estimated low-flows using LMO and PWM of WB2 are minimum when compared with those values of MLM and PME.
- CC values indicated that there is generally good correlation between the observed and estimated low-flows and these values vary between 0.979 and 0.988.

Recommendation

For river Godavari at Polavaram gauging site, the estimated low-flows by WB2 (using LMO or PWM) distribution could be used for water quality management, reservoir storage design, safe surface water withdrawals deciding environmental flows and minimum water release policy and so on.

THANK YOU